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16/ENG04/050
ENG 381
Elect/Elect

1. $y = e^{x^2+x}$

Let $x^2 + x = u$

$\frac{du}{dx} = 2x + 1$

$y = e^u \quad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^u (2x+1)$

$= (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^u + e^{x^2+x}(2)$

Recall $y = e^{x^2+x}$

$y' = (2x+1)e^u$

$\therefore y'' = (2x+1)y' + 2y$

From y''

$y^{(n)} = (2x+1)y^{(n-1)} + 2(n-1)y^n$

$\therefore y^{(n+2)} = (2x+1)y^{(n+2-1)} + 2(n+2-1)y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2 i $y = x^3 e^{4x}$

$\sum_{r=0}^{4} {}^nC_r u^{(n-r)} v^r$

$v = x^3, v^{(1)} = 3x^2, v^{(2)} = 6x, v^{(3)} = 6, v^{(4)} = 0, v^{(5)} = 0$

$u = e^{4x}, u^{(1)} = 4e^{4x}, u^{(2)} = 16e^{4x}, u^{(3)} = 64e^{4x}, u^{(4)} = 256e^{4x},$

$u^{(5)} = 1024e^{4x}$

$y^5 = u^{(5)}v + 5u^{(5-1)}v^{(1)} + {}^5C_2 u^{(5-2)}v^{(2)} + {}^5C_3 u^{(5-3)}v^{(3)} +$
${}^5C_4 u^{(5-4)}v^{(4)}$

$= u^{(5)}v + 5u^{(4)}v^{(1)} + {}^5C_2 u^{(3)}v^{(2)} + {}^5C_3 u^{(2)}v^{(3)} + {}^5C_4 u^{(1)}v^{(4)}$

$= 1024e^{4x}x^3 + 5(256e^{4x})(3x^2) + 10(64e^{4x})6x +$
$10(16e^{4x})6 + 5(4e^{4x})(0)$

$= 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x} + 0$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y' = 0$

$x^2y'' = w_1$

$v = x^2,\ v^{(1)} = 2x,\ v^{(2)} = 2$

$u = y'',\ u^{(1)} = y^{(3)},\ u^{(2)} = y^{(4)},\ u^{(n)} = y^{(n+2)}$

$w_1^{(n)} = \sum_{r=0}^{2} {}^nC_r u^{(n-r)}v^r$

$= u^{(n)}v + nu^{(n-1)}v^{(1)} + {}^nC_2 u^{(n-2)}v^{(2)} \cancel{= u^{(n)}v + nu^{(n-1)}v^{(1)}} +$
$\cancel{n(n-1)}$

$= u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}}{2}v^{(2)}$

$= y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)y^n(2)}{2}$

$w_1^{(n)} = y^{(n+2)}x^2 + ny^{(n+1)}2x + n(n-1)y^n$

$xy' = w_2$

$v = x,\ v^{(1)} = 1$

$u = y^{(1)},\ u^{(1)} = y^{(2)}\quad u^n = y^{(n+1)}$

$w_2 = u^{(n)}v + nu^{(n-1)}v^{(1)}$

$= y^{(n+1)}x + ny^{(n)}$

$w_3 = y'$

$v = 1$

$u = y'$

$u^{(n)} = y^{(n)}$

$w_3 = u^{(n)} v$

$w_3 = y^n$

$w_1^{(n)} + w_2^{(n)} + w_3^{(n)} = 0$

$x^2 y^{(n+2)} + n y^{(n+1)} 2x + n(n-1) y^n + y^{(n+1)} x + n y^{(n)} + y^{(n)} = 0$

$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n(n-1) + n + 1) y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 - n + n + 1) y^{(n)} = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$