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16/ENG04/001
ELECTRICAL/ELECTRONICS ENGINEERING
ENGINEERING MATHEMATICS III [ENG 381] ASSIGNMENT II
300LEVEL

Problem Statements

1. If $y = e^{x^2+x}$,

Show that

$$y'' = y'(2x+1) + 2y$$

and hence Prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$$

2. Using the Leibnitz theorem, given that

i $y = x^3 e^{4x}$, determine $y^{(5)}$

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

SOLUTION

1 If $y = e^{x^2+x}$, showing $y'' = y'(2x+1) + 2y$

$$y = e^{x^2+x}$$

$$y' = \frac{dy}{dx} = (2x+1)e^{x^2+x}$$

To solve ~~for~~ y'' from y'. Let $(2x+1) = u$ and $e^{x^2+x} = v$

$\therefore$ $u = 2x+1$ $\qquad$ $v = e^{x^2+x}$

$du/dx = 2$ $\qquad$ $dv/dx = (2x+1)e^{x^2+x}$

Applying Product Rule to ~~eqn~~ y' to get y''

$$u\,dv/dx + v\,du/dx$$

$$\therefore y'' = \frac{d^2y}{dx^2} = u\,dv/dx + v\,du/dx$$

$$y'' = 2x+1(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

but; $y = e^{x^2+x}$

$y' = 2x+1(e^{x^2+x})$

Hence $\Rightarrow$ $y'' = y'(2x+1) + 2y$ //

Proof that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

from

$$y'' = y'(2x+1) + 2y$$

equating to zero (0)

$$y'' - y'(2x+1) - 2y = 0$$

Applying Leibnitz theory

$$\underset{W_1}{y''} - \underset{W_2}{y'(2x+1)} - \underset{W_3}{2y} = 0$$

$$\boxed{W = \sum_{r=0}^{n} {}^{n}C_r\, u^{(n-r)} v^{(r)}}$$

$W_1 = \underset{u}{y''} \cdot \underset{v}{1}$

$u = y''$ $\qquad$ $v = 1$

$u = u^{(0)} = y^{(2)}$ $\qquad$ $v = v^{(0)} = 1$

$u^{(1)} = y^{(3)}$ $\qquad$ $v^{(1)} = 0$

$u^{(2)} = y^{(4)}$

$u^{(n)} = y^{(n+2)}$

$$W_1 = \sum_{r=0}^{n} {}^{n}C_r\, u^{(n-r)} v^{(r)}$$

$$W_1 = u^{(n)}v^{(0)} + nu^{(n-1)}v^{(1)} = ny^{(n-1)} \cdot 0$$

$$W_1^{n} = y^{(n+2)} \cdot 1$$

$$W_1^{n} = y^{(n+2)}$$

$W_2 = \underset{u}{y'}\,\underset{v}{(2x+1)}$

$u = y'$ $\qquad$ $v = 2x+1$

$u^{(1)} = y^{2}$ $\qquad$ $v' = 2$

$u^{(2)} = y^{3}$

$u^{n} = y^{(n+1)}$

$$W_2 = u^{n}v + nu^{(n-1)}v' + \cancel{n(n-1)}$$

$$W_2^{n} = y^{(n+1)}(2x+1) + ny^{(n)}(2)$$

$$= (2x+1)y^{(n+1)} + 2ny^{(n)}$$

$W_3 = \underset{v}{2}\,\underset{u}{y}$

$u = y$ $\qquad$ $v = 2$

$u^n = y^n$ $\qquad$ $v' = 0$

$W_3 = \sum_{r=0}^{n} {}^nC_r\, u^{(n-r)} v^{(r)}$

$W_3 = u^n v + \cancel{n u^{(n-1)} v^{(1)}}^{\,0}$

$W_3^n = y^{(n)} \cdot 2 = 2y^n$

from

$y'' - y'(2x+1) - 2y = 0$

Also

$\Rightarrow W_1^n - W_2^n - W_3^n = 0$

$W_1 = y^{(n+2)}$

$W_2^n = (2x+1)y^{(n+1)} + 2ny^{(n)}$

$W_3^n = 2y^n$

$\therefore y^{(n+2)} - [(2x+1)y^{(n+1)} + 2ny^{(n)}] - 2y^n = 0$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1)$

$\underline{y^{(n+1)} = (2x+1)y^{(n+1)} + 2(n+1)y^n}$

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No. 2

i Using the Leibnitz theorem, given that $y = x^3 e^{4x}$, determine $y^{(5)}$

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that $x^2 y^{(n+2)}$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

i $y = x^3 e^{4x}$, determining $y^{(5)}$

$\downarrow v \quad \downarrow u$

$u = e^{4x}$ $\qquad$ $V = x^3$

$u' = 4e^{4x}$ $\qquad$ $V' = 3x^2$

$u'' = 16e^{4x}$ $\qquad$ $V'' = 6x$

$u''' = 64e^{4x}$ $\qquad$ $V''' = 6$

$u^n = 4^n e^{4x}$ $\qquad$ $V^{iv} = 0$

$$y^n = \sum_{r=0}^{n} {}^nC_r\, u^{(n-r)}\, v^{(r)}$$

$$y^n = {}^nC_0 u^n v^0 + {}^nC_1 u^{(n-1)} v^{(1)} + {}^nC_2 u^{(n-2)} v^2 + {}^nC_3 u^{(n-3)} v^3$$

$$= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)u^{(n-3)}}{2\times 3} v^3$$

$$= 4^n e^{4x} . x^3 + n(4^{n-1}e^{4x}) \cdot 3x^2 + \frac{n(n-1)}{2}[4^{n-2}e^{4x}] \cdot \overset{3}{\cancel{6}}x$$
$$+ \frac{n(n-1)(n-2)}{6} \cdot 4^{n-3} e^{4x} \cdot \cancel{6}$$

$$= 4^n x^3 e^{4x} + 3nx^2[4^{n-1}e^{4x}] + n(n-1)[4^{n-2}e^{4x}] \cdot 3x$$
$$+ [n^3 - 3n^2 + 2n] 4^{n-3} e^{4x}$$

Hence for y^5

$$y^5 = 4^5 x^3 e^{4x} + 3(5)x^2[4^{(5-1)}e^{4x}] + 5(5-1)[4^{(5-2)}e^{4x}] \cdot 3x$$
$$+ [5^3 - 3(5)^2 + 2(5)] 4^{(5-3)} e^{4x}$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + \cancel{1280}\ \overset{1280}{\cancel{3840}}x\, e^{4x} + 960e^{4x}$$
$$= 64e^{4x}[16x^3 + 60x^2 + 20x + 15]$$

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b. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' = w_1$

$v = x^2$ $\quad u = y'' = y^2$

$v' = 2x$ $\quad u' = y''' = y^{(3)}$

$v'' = 2$ $\quad u'' = y^{iv} = y^{(4)}$

$u^n = y^{n+2}$

$W_1^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2}u^{n-2}v^2$

$W_1^n = y^{n+2}\cdot x^2 + n(y^{n+2-1})\cdot 2x + n(n-1)y^{n+2-2}\cdot x$

$W_1^n = x^2(y^{n+2}) + 2nx(y^{n+1}) + n(n-1)y^n$

$W_2 = xy'$

$u = y'$ $\quad v = x$

$u' = y'' = y^2$ $\quad v' = 1$

$u^n = y^{n+1}$

$W_2^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2}u^{n-2}v^2$

$= (y^{n+1})x + nu^{n-1}v' + \frac{n(n-1)}{2}u^{n-2}v^2$

$W_2^n = (y^{n+1})x + n(y^{n+1-1})\cdot 1$

$= xy^{n+1} + ny^n$

$W_3 = y$

$u = y$ $\quad v = 1$

$u' = y'$ $\quad v' = 0$

$u^n = y^n$

$W_3^n = u^n v^0 + nu^{n-1}v'$

$W_3^n = y^n\cdot 1 + ny^{n-1}\cdot 0$

$W_3^n = y^n$

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From equation $W_1^n + W_2^n + W_3^n$

$\therefore \quad x^2 y^{n+2} + 2nx(y^{n+1}) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

factorizing

$x^2 y^{n+2} + 2nx(y^{n+1}) + xy^{n+1} + n(n-1)y^n + ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$\therefore$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

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