Ujunwa Fatima
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Computer Engineering
ENG 381

Solution

$y = e^{x^2+x}$

let $p = x^2+x$ $y = e^p$

$\frac{dp}{dx} = 2x+1$; $\frac{dy}{dp} = e^p$

$\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$

$\frac{dy}{dx} = e^p \times 2x+1$

$\frac{dy}{dx} = e^{x^2+x} \times 2x+1$

$\frac{dy}{dx} = 2x+1 \left(e^{x^2+x}\right)$

$y' = 2x+1 \left(e^{x^2+x}\right)$

let $y' = u$

Taking second derivative using product rule

$U\frac{dv}{dx} + V\frac{du}{dx}$

$y'' = (2x+1)\left(2x+1\, e^{x^2+x}\right) + \left(e^{x^2+x}\right)(2)$

$y'' = 2\left(e^{x^2+x}\right) + (2x+1)(2x+1)e^{x^2+x}$

recall that $y = e^{x^2+x}$ and $y' = (2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

a) $z_1 = y^{(2)}$

$z_2 = y^{(1)}$

$z_3 = 2y$

let $u = y^{(2)}$, $v = 1$
$u^n = y^{n+2}$, $v' = 0$
hence, $W_1^n = u^n v^2 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-1} v^2$

$W_1^n = y^{n+2} \cdot 1 + n y^{n-1+2} \cdot 2$
$W_1^n = y^{n+2}_{(1)}$

b) $W_2 = y' (2x+1)$
$u = y'$ $v = 2x+1$
$u' = y''$, $v' = 2$, $v'' = 0$
$u^n = y^{n+1}$
$W_2^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$
$w_2^n = y^{n+1} (2x+1) + n(y^{n+1-1}) 2 + \frac{n(n-1)}{2} y^{n-2+1}$

$W_2^n = (2x+1) y^{n+1} + 2n y^n$
$W_3 = 2y$
$u = y$ $v = 2$
$u' = y'$ $v' = 0$
$u^n = y^n$
$W_3^n = y^n \cdot 2 + n y^{n-1} \cdot 0$
$W_3^n = 2y^n$
$W_1^n = W_2^n + W_3^n$
$y^{(n+2)} = (2x+1) y^{(n+1)} + 2n y^n + 2y^n$ (it is proven)

2.
$y = x^3 e^{4x}$
$v = x^3$, $u = e^{4x}$
$v' = 3x^2$ $u' = 4e^{4x}$ $\therefore u^n = 4^n e^{4x}$
$v'' = 6x$, $u'' = 16e^{4x}$
$v''' = 6$, $u''' = 64e^{4x}$
$y^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} v^2 u^{n-2} + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

if $n = 5$
$y^{(5)} = (4^5 e^{4x} \cdot x^3) + (5 \cdot 4^{5-1} e^{4x} \cdot 3x^2) + \frac{5(5-1)}{2 \times 1} 4^{5-2} e^{4x} \cdot 6x$

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$$+\frac{5(5-1)(5-2)4^{5-3}e^{4x}\cdot 6}{3\times 2\times 1}$$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 2880e^{4x}x + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 160x + 15)$$

$$y$$

" $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

(a) $W_1 = x^2y''$ $y^{(n)}$

$v = x^2,\ u = y''$

$v' = 2x\quad u' = y'''$

$v'' = 2,\ u'' = y^{iv}$

$u^n = y^{n+2}$

$W_1^n = y^{n+2}\cdot x^2 + n(y^{n+2-1})2x + n(n-1)y^{n+2-2}\cdot x$

$W_1^n = x^2(y^{n+2}) + 2nx(y^{n+1}) + n(n-1)y^n 2$