

$$2) \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\Rightarrow \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$x = \nu e^{kt}$$

$$\frac{dx}{dt} = k\nu e^{kt} = kx$$

$$\frac{d^2x}{dt^2} = k^2\nu e^{kt} = k^2x$$

Substitute for  $\frac{dx}{dt}$ ,  $\frac{d^2x}{dt^2}$  &  $x$  into

$$k^2x + 5kx + 6x = 0$$

divide through by  $x$

$$k^2 + 5k + 6 = 0 \rightarrow \text{Auxiliary equation}$$

$$(k^2 + 3k) + (2k + 6) = 0$$

$$k(k+3) + 2(k+3) = 0$$

$$(k+3)(k+2)$$

$$k = -3 \text{ or } -2$$

$$x = Ae^{-3t} + Be^{-2t} \rightarrow \text{General solution}$$

taking the standard form of the RHS

$$x = A\cos t + B\sin t$$

$$\frac{dx}{dt} = -A\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$$

Subst for  $\frac{dx}{dt}$ ,  $\frac{d^2x}{dt^2}$  &  $x$

$$[-A\cos t - B\sin t] + 5[-A\sin t + B\cos t] + 6[A\cos t + B\sin t] = \cos t$$

$$-A\cos t - B\sin t + -5A\sin t + 5B\cos t + 6A\cos t + 6B\sin t = \cos t$$

$$\sin t (-5A + 5B) = \cos t (5B - 5A + 1)$$

$$\sin t (-5A + 5B) + \cos t (5B + 5A) = \cos t$$

Equate the coefficients

$$\rightarrow -5A + 5B = 0 \quad \text{--- (i)}$$

$$\ominus \underline{5A + 5B = 1} \quad \text{--- (ii)}$$

$$-10A = -1$$

$$A = \frac{1}{10}$$

$$-5A + 5B = 0$$

$$5B = 5A$$

$$B = \frac{5A}{5}$$

$$B = A = \frac{1}{10}$$

Particular Integral  $\rightarrow x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

$$x = \frac{1}{10} \int (\cos t + \sin t)$$

~~Complete general solution  $\rightarrow Ae^{-3t} + B$~~

Recall! General solution  $\rightarrow x = Ae^{-3t} + Be^{-2t}$

When  $t=0$ ;  $x=0.1$  &  $\frac{dx}{dt} = 0$        $\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t}$

substituting  $x$  &  $t$

$$0.1 = Ae^{-3(0)} + Be^{-2(0)}$$

$$0.1 = A + B \quad \text{--- (i)}$$

substituting  $\frac{dx}{dt}$  &  $t=0$

$$0 = -3Ae^{-3(0)} - 2Be^{-2(0)}$$

$$0 = -3A - 2B \quad \text{--- (ii)}$$

$$A = 0.1 - B \quad \text{--- (iii)}$$

$$0 = -3(0.1 - B) - 2B$$

$$0 = -0.3 + 3B - 2B$$

$$0.3 = 3B - 2B$$

$$B = 0.3$$

From eq. (ii)

$$A = 0.1 - AB - 2B$$

$$A = 0.1 - 0.3 - 2(0.3)$$

$$A = -0.2$$

$$\therefore \text{General solution} \rightarrow x = -0.2e^{-3t} + 0.3e^{-2t}$$

$$\text{Complete general solution} \rightarrow x = -0.2e^{-3t} + 0.3e^{-2t} + \frac{1}{10} [\cos t + \sin t]$$

100) steady state equation

$$x = \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{a}\right)$$