

16 / ENCG-06 / 041

$$y' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x} \quad (2)$$

$$y'' = y'(2x+1) + 2y$$

From Leibnitz Theorem

$$y'' - y'(2x+1) - 2y = 0$$

$$u_1 = y''$$

$$v = y''$$

$$v = 1$$

$$v' = 0$$

$$u^n = y^{n+2}$$

$$m_i = \binom{n}{2} u^{n-2} v^2$$

$$= y^{n+2}$$

$$u_2 = y'(2x+1)$$

$$v = y'$$

$$u^n = y^{n+1}$$

$$v' = 2$$

$$v = 2x+1$$

$$v' = 2$$

$$v^n = \binom{n}{0} u^{n-0} v^0 + \binom{n}{1} u^{n-1} v^1$$

$$y'' - 2y'(2x+1) - 2y = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

2.

$$y = x^3 e^{4x}$$

$$v = x^3, v' = 3x^2, v'' = 6x, v''' = 6$$

$$u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u''' = 64e^{4x}$$

$$y^n = {}^n C_0 u^{n-0} v^0 + {}^n C_1 u^{n-1} v^1 + {}^n C_2 u^{n-2} v^2 + {}^n C_3 u^{n-3} v^3$$

$$= u^n v^0 + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$$

$$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$\frac{4^{n-3} e^{4x} \cdot 6}{6}$$

$$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

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$$= 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2)]$$

$$= 4^{n-3} e^{4x} [64x^3 + n 48x^2 + 12(n-1)x + n(n-1)(n-2)]$$

$$y^5 = 4^{5-3} e^{4x} [64x^3 + (5 \times 48)x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2)]$$

$$x^2 y'' + xy' + y = 0$$

$$w_1 + w_2 + w_3 = 0$$

$$y^n [y^2 x^2 + n 2xy + n(n-1)] + y^n (xy + n) + y^n = 0$$

⊙ At  $x=0$

$$y^n (n(n-1) + n) + y^n = 0$$

$$n(n-1) + n + 1 = 0$$

$$y^n = -y^n (n-1) - ny^n$$

$$\text{at } n=1$$

$$y = -0 - y'$$

$$y = -y'$$

$$\Rightarrow x^2 y^{n+2} + n 2xy^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$\Rightarrow x^2 y^{n+2} + 2xy^{n+1} (n+1) + y^n (n^2 - n + n + 1) = 0$$

$$ii. \quad x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

$$W_1 = x^2 y''$$

$$U^0 = U = y''$$

$$U' = y''', \quad U'' = y''''$$

$$U^n = y^{n+2}$$

$$V = x^2, \quad V' = 2x, \quad V'' = 2$$

$$V''' = 0$$

$$W_2 = x y'$$

$$U^0 = U = y'$$

$$U' = y'', \quad U'' = y'''$$

$$U^n = y^{n+1}$$

$$V = x, \quad V' = 1, \quad V'' = 0$$

$$W_3 = y$$

$$U = y$$

$$U' = y', \quad U'' = y''$$

$$U^n = y^n$$

$$V = 1, \quad V' = 0$$

$$W_1^n = {}^n C_0 U^{n-0} V^0 + {}^n C_1 U^{n-1} V^1 + {}^n C_2 U^{n-2} V^2$$

$$= U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V''$$

$$= y^{n+2} V + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n x^2$$

$$= y^{n+2} V + n y^{n+1} 2x + (n-1)n y^n$$

$$= y^n [y^2 x^2 + n y^2 x + n(n-1)]$$

$$W_2^n = {}^n C_0 U^{n-0} V^0 + {}^n C_1 U^{n-1} V^1 + {}^n C_2 U^{n-2} V^2 + {}^n C_3 U^{n-3} V^3$$

$$= U^n V' + n U^{n-1} V'' + \frac{(n-1)n}{2!} U^{n-2} V''' = 0$$

$$= y^{n+1} \cdot x + n y^n \cdot 1 + 0$$

$$= y^n (xy + n)$$

$$W_3^n = {}^n C_0 U^{n-0} V^0 + {}^n C_1 U^{n-1} V^1$$

$$= U^n V^0 + 0$$

$$= 1^n$$