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ENG 381 Assignment
PETROLEUM ENGINEERING

1) If $y = e^{x^2+x}$

Show that

$y'' = y'(2x+1) + 2y$ and hence, prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution.

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = (2x+1)y' + 2y$

$\therefore y'' = y'(2x+1) + 2y$

Using Leibnitz theorem

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \ldots$

from $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Let $A = y''$

$u^n = y''$

$u^n = y^{n+2}$

Let $B = y'(2x+1)$

$u = y'$ $\quad$ $v = 2x+1$

$u^n = y^{n+1}$ $\quad$ $v' = 2$

$v'' = 0$

$y^n = y^{n+1}(2x+1) + ny^{n+1-1}(2) + \frac{n(n-1)}{2!}y^{n+1-2}(0)$

$y^n = y^{n+1}(2x+1) + 2ny^n$

Let $C = 2y$

$u = y$

$y^n = y^n$

$u^n = y^n(2) + 2$

$u^n = 2y^n$

$y^n = A - B - C$

$y^{n+2} - (y^{(n+1)}(2x+1) + 2nxy^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2nxy^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2nxy^n + 2y^n$

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)xy^n$

2) Using the Leibnitz theorem given that

i) $y = x^3 e^{4x}$, determine $y(5)$

ii) $\frac{x^2 d^2y}{dx^2} + \frac{x\,dy}{dx} + 4y = 0$

Solution:

i) $y = x^3 e^{4x}$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \dots$

let $v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x}$

$v''' = 6$ $\quad u''' = 64e^{4x}$

$v'''' = 0$ $\quad u'''' = 256e^{4x}$

$v''''' = 0$ $\quad u''''' = 1024e^{4x}$

$y^5 = u^5 v + 5u^{5-1}v' + \frac{5(5-1)}{2!}u^{5-2}v^2 + \frac{5(5-1)(5-2)}{3!}u^{5-3}v^3 +$

$\frac{5(5-1)(5-2)(5-3)}{4!}u^{5-4}v^4 + \frac{5(5-1)(5-2)(5-3)(5-4)}{5!}u^{5-5}v^5$

$y^5 = u^5 v + 5u^4v' + 10u^3v^2 + 10u^2v^3 + 5u'v^4 + uv^5$

$y^5 = 1024e^{4x}x^3 + 5(256e^{4x})3x^2 + 10(64e^{4x})6x + 10(16e^{4x})6$
$+ 5(4e^{4x})(0) + e^{4x}(0)$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x} + 0 + 0$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

ii) $\frac{x^2 d^2y}{dx^2} + \frac{x\,dy}{dx} + 4y = 0$

Using Leibnitz theorem

$x^2y'' + xy' + 4y = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}}{2!}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \dots$

let $A = x^2 y''$

$u = y''$ $\qquad$ $v = x^2$

$u^n = y^{n+2}$ $\qquad$ $v' = 2x$

$v'' = 2$

$v''' = 0$

$$y^n = y^{n+2} x^2 + n y^{n+2-1} 2x + \frac{n(n-1)}{2!} y^{n+2-2}(2) + \frac{n(n-1)(n-2)}{3!}$$

$$y^{n+2-3}(0)$$

$$y^n = y^{n+2} x^2 \; n y^{n+1} 2x + n(n-1) y^n$$

let $B = x y''$

$u = y'$ $\qquad$ $v = x$

$u^n = y^{n+1}$ $\qquad$ $v' = 1$

$v'' = 0$

$$y^n = y^{n+1} x + n y^{n+1-1} (1) + \frac{n(n-1)}{2!} y^{n+1-2}(0)$$

$$y^n = y^{n+1} x + n y^n + 0$$

$$y^n = y^{n+1} x + n y^n$$

let $C = y$

$u^n = y^n$

$y^n = A + B + C$

$$y^n = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1) y^n + y^{n+1} x + n y^n + y^n$$

$$y^n = y^{n+2} x^2 + y^{n+1} (2nx + x) + y^n (n(n-1) + n + 1)$$

$$y^n = y^{n+2} x^2 + y^{n+1} x (2n+1) + y^n (n^2 - n + (n+1))$$

$$y^n = y^{n+2} x^2 + y^{n+1} x (2n+1) + y^n (n^2+1) = 0$$