$\omega_3 = (-2x+1)y'$

$u^0 = y'$

$u^n = y^{n+1}$

$v = -2x+1$

$v' = -2$

$\omega_3 = u^n v + n u^{(n-1)} v'$

$= y^{n+1}(-2x+1) + n y^{n+1-1}(-2)$

$= -2x+1(y^{n+1}) - 2ny^n$

$\omega_1 + \omega_2 + \omega_3$

$= y^{n+2} + (-2y^n) + (-2x+1)(y^{n+1}) - 2ny^n$

$= y^{n+2} - 2y^n(1+n) - y^{n+1}(2x+1)$

$y^{n+2} = 2y^n(1+n) + y^{n+1}(2x+1)$

$v = x \qquad u = y'$

$v' = 1 \qquad u' = y'' \,, \quad u^n = y^{n+1}$

$w' = u^n v^0 + n u^{n-1} v' + (n-1) n\, u^{n-2} v'' + \cdots$

$w' = y^{n+1} x + n y^n + (n-1)(n) u^{n+1-2}$

$= y^{n+1} x + n y^n$

also,

$w = y$

$v = 1 \,, \quad u = y$

$u^n = y^n$

$w = u^n v^0$

$= y^n \cdot 1 = y^n$

$w = y^{n+2} x^2 + n y^{n+1} 2x + (n-1) n y^n + y^{n+1} x + n y^n + y^n$

$w = y^n (n^2 - n + n + 1) + y^{n+1} (n 2x + x) + x^2 y^{n+2}$

$w = y^n (n^2 + 1) + x y^{n+1} (2n+1) + x^2 y^{n+2}$

$x^2 y^{(n+2)} + (2n+1) x y^{n+1} + (n^2+1) y^n$ *

$u = e^{4x}$

$u' = 4e^{4x}$

$u'' = 16e^{4x}$

$u^n = 4^n e^{4x}$

$y^n = u^n v^0 + nu^{n-1}v^1 + (n-1)nu^{n-2}v'' + \frac{(n-2)(n-1)nu^{(n-3)}}{3}v'''$

$y^n = 4^n e^{4x}x^3 + n4^{n-1}e^{4x}3x^2 + \frac{(n-1)(n)}{2}4^{n-2}e^{4x}6x + \frac{(n-2)(n-1)n}{3\times 2}$

$4^{(n-3)}e^{4x}6$

$y^n = 4^n e^{4x}x^3 + n4^{n-1}e^{4x}3x^2 + (n-1)n4^{n-2}e^{4x}3x + \cancel{(n-2)(n-1)n4^{n-3}e}$

$(n-2)(n-1)n4^{n-3}e^{4x}$

~~If the~~

$y^5 = 4^5 e^{4x}x^3 + 5(4^{5-1})e^{4x}3x^2 + (5-1)(5)4^{(5-2)}e^{4x}3x +$

$(5-2)(5-1)(5)4^{(5-3)}e^{4x}$

$y^5 = 1024e^{4x}x^3 + 3840x^2e^{4x} + 3840e^{4x} + 960^{4x}$

ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

If $x^2\frac{d^2y}{dx^2} = w''$, $x\frac{dy}{dx} = w'$, $y = w$

then $w = w'' + w' + w$

Hence,

$w'' = x^2\frac{d^2y}{dx^2}$

$u = \frac{d^2y}{dx^2} = y''$, $u' = y'''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$

soln.

$w'' = u^n v^0 + nu^{n-1}v' + \frac{(n-1)(n)}{2}u^{n-2}v'' + \ldots$

$w'' = y^{n+2}x^2 + ny^{n+1}2x + (n-1)(n)y^n$

also,

$w' = x\frac{dy}{dx}$

1) If $y = e^{x^2+x}$

$u = x^2 + x$

$\frac{du}{dx} = 2x + 1$

$y = e^u$

$\frac{dy}{du} = e^u$

$$\frac{dy}{dx} = \left(e^{x^2+x}\right)(2x+1) =$$

$u = e^{x^2+x}$, $v = 2x+1$

$$\frac{d^2y}{dx^2} = e^{x^2+x}(2) + (2x+1)\left(e^{x^2+x}\right)(2x+1)$$

If $e^{x^2+x} = y$

$$y'' = 2x + (2x+1)\frac{dy}{dx}$$

$$y'' = 2x + (2x+1)y'$$

$w_1 = 1\,\frac{d^2y}{dx^2}$

$v = 1$, $v' = 0$

$u = y''$, $u^n = y^{n+2}$

$= u^n v^0 + n u^{n-1} v'$

$= y^{n+2} 1 + n y^{n+2-1}$

$= y^{n+2}$

b) $w_2 = -2y$

$v = -2$, $v' = 0$

$u = y$, $u^n = y^n$

$= u^n v^0 + n u^{n-1} v'$

$= \cancel{y^n}\, -2y^n + n y^{n-1} \cdot 0$

$= -2y^n$

2) $y = x^2 e^{4x}$

$v = x^3$

$v' = 3x^2$

$v'' = 6x$

$v''' = 6$