ENG 381
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Computer Engineering

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

Where: $u = 2x+1$ $\quad v = e^{x^2+x}$

$\frac{dy}{dx} = 2 \quad \frac{du}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{du}{dx} + v\frac{dv}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$\therefore y'' = y'(2x+1) + 2y$

from Leibnitz theorem

$y'' - y'(2x+1) - 2y = 0$

$1 = y'' \quad v = 1 \quad u^n = y^{n+2}$

$u = y'' \quad v' = 0 \quad m_1^n = {}^nC_2 u^{n-0} v$

$= y^{n+2}$

$w_2 = y'(2x+1) \quad v = 2x+1$

$u = y' \quad v' = 2$

$u^n = y^{n+1} \quad v^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$v'' = 0$

$= u^n v + n v^{n-1}$

$= y^{n+1}(2x+1) + n y^n 2$

$w_3 = 2y$

$u = y$

$u^n = y^n \quad v = 2 \quad v' = 0$

$w_3^n = {}^nC_0 u^{n-0} v$

$= 2y^n$

$y^n - y'(2x+1) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

$u^n = y\,n+1$

$w_3 = y$

$u = y$ $\quad v = 1 \quad v' = 0$

$u' = y', \; u'' = y''$

$u^n = y^n$

$w_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v''$

$= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

$= y^{n+2} v + n y^{n+1}\, 2x + \frac{n(n-1)}{2} y^n 2$

$= y^{n+2} v + n y^{n+1}\, 2x + (n-1) n\, y^n$

$= y^n [y^2 x^2 + n y^2 + n(n-1)]$

$w_2^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v'' + {}^nC_3$
$u^{n-3} v'''$

$= u^n v^0 + n u^{n-1} v' + \frac{(n-1)n}{2!} u^{n-2} \cdot 0$

$= y^{n+1} x + n y^n \cdot 1 + 0$

$= y^n (xy + n)$

$w_3^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^n v^0 + 0$

$= y^n$

$x^2 y'' + x y' + y = 0$

$w_1 + w_2 + w_3 = 0$

$y^n (y^2 x^2 + n 2xy + n(n-1)] + y^n (xy + n) + y^n = 0$

$At \quad x = 0$

$y^n n(n-1) + y^n n + y^n = 0$

$n(n-1) + n + 1 = 0$

$y^n = -y^n n(n-1) - n y^n$

$a + n = 1$

$y = -0 - y'$

$y = -y'$

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2) $y = x^3 e^{4x}$

$V^0 = x^3$, $V' = 3x^2$, $V'' = 6x$ $V''' = 6$

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$

$u^n = 4^n e^{4x}$

$y^n = {}^nC u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC u^{n-2} v'' + {}^nC u^{n-3} v'''$

$= v^n v' + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u'''$

$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1) 4^{n-2} e^{4x}}{7}$

$+ (n-1)(n-2) \frac{4^{n-3} e^{4x} \cdot 6^{x}}{3 \times 2}$

$4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$= 4^{n-3} e^{4x} ( 4^3 x^3 + n 4^2 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2) ]$

$= 4^{n-3} e^{4x} [ 64x^3 + n 48 x^2 + 12(n-1) + n(n-1)(n-2) ]$

$y^5 = 4^{5-3} e^{4x} [ 64x^3 + (5 \times 48) x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2)$

$y^5 = 16 e^{4x} [ 64x^3 + 240x^2 + 240x + 60 ]$

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$w_1 = x^2 y''$

$u^0 = v - y''$

$v' = y''' \; y'' = y^{iv}$

$u^n = y^{n+2}$

$w_2 = x y'$

$u^0 = u = y'$ $\qquad V = x$, $V' = 1$, $V'' = 0$

$v' = y''$ $u'' = y'$

$u' = y''$ $v'' = y'''$

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$= x^2 y_{n+2} + n 2x y_{n+1} + n(n-1) y_n + x y_{n+1} + n y_n + y_n = 0$

$= x^{1}{}_{n+2} + x y_{n+1} (2n+1) + y_n (n^2 - n + n + 1) = 0$

$= x^{1} y_{n+2} + (2n+1) x y_{n+1} + (n^2+1) y_n = 0$