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1) $y'' = y'(2x+1) + 2y$

For $y'' \Rightarrow y^{(n+2)} + ny^{(n+1)} \cdot 0$
$= y^{(n+2)}$

For $y'(2x+1)$; $y' = u$
$(2x+1) = v$

$y'(2x+1) = y^{(n+1)}(2x+1) + ny^{n}(2)$

For $2y \Rightarrow 2y^n$

$\therefore$ We have;

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$

$\Rightarrow y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) (i) $y = x^3 e^{4x}$

$x^3 = v$
$e^{4x} = u$

$u = e^{4x}$; $v = x^3$
$u' = 4e^{4x}$; $v' = 3x^2$
$u'' = 16e^{4x}$; $v'' = 6x$
$u''' = 64e^{4x}$; $v''' = 6$
$u'''' = 256e^{4x}$; $v^{(4)} = 0$
$u^{(5)} = 1024^{4x}$,

$y^{(5)} = u^{(5)}v + 5u^{(4)}v^{(1)} + 10u^{(3)}v^{(2)} + 10u^{(2)}v^{(3)} + 5u^{(1)}v^{(4)} + uv^{(5)}$

$y^{(5)} = 1024e^{4x} \cdot x^3 + [5(256e^{4x} \cdot 3x^2)] + [10(64e^{4x} \cdot 6x)] + [10(16e^{4x} \cdot 6)$
$+ [5(4e^{4x} \cdot 0)] + 0.$

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$y^{(5)} = 1024e^{4x} \cdot x^3 + 3840e^{4x} + 3840e^{4x} + 960e^{4x}$

$= e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

$\underline{y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)}$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

$= x^2y'' + xy' + y = 0.$

Using Leibnitz Theorem.

from x^2y'', $u = y''$, $x^2 = v$

$x^2y'' = {}^nC_0 u^{(n)} v + {}^nC_1 u^{(n-1)} v^{(1)} + {}^nC_2 u^{(n-2)} v^{(2)}$

$= y^{(n+2)}x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{\cancel{2}!} y^{(n)} \cdot \cancel{2}$

$x^2y'' = y^{(n+2)} \cdot x^2 + 2xny^{(n+1)} + n(n-1)y^{(n)}$

for $xy' = u = y'$ and $v = x$.

$xy' = y^{(n+1)}x + ny^{(n)}$

For $y = y^{(n)}$

$\therefore$ We have;

$y^{(n+2)}x^2 + ny^{(n+1)}2x + n(n-1)y^{(n)} + y^{(n+1)}x + ny^{(n)} + y^{(n)}$

$y^{(n+2)}x^2 + xy^{(n+1)}(2n+1) + y^{(n)}[n(n-1)(n+1)]$

$y^{(n+2)}x^2 + xy^{(n+1)}(2n+1) + y^{(n)}[n(n^2-1)]$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^3-n)$

$= \underline{x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^3-n)}$