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161ENG02/006.

Computer Eng.

... theorem Rule / Function of a function.

$u = e^u$ $\quad y = e^u$

$u = x + x^2$ $\quad {}^{dy}/_{du} = e^u$

$\frac{du}{dx} = 1 + 2x$

$dy/du \cdot du/dx = (2x+1)e^u$

${}^{dy}/_{dx} = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x+x^2})$

$y'' = \left(\frac{d^2y}{dx^2}\right) = [2x+1][(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

$y'' = (2x+1)[(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$\therefore y'' = y'(2x+1) + 2(y)$

(b.) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$W_1{}^n = y''$ $\quad v_1 = 1$ $\quad U^0 = y''$

$\quad v_1 = 0$ $\quad U^n = y^{n+2}$

$W_1{}^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' +$

$W_1{}^n = y^{n+2}(1) + ny^{n-1}(0) + \ldots$

$\therefore W_1{}^n = y^{n+2}$

Expanding $y'(2x+1)$

$W_2^n = 2xy'$

$V^{(0)} = -2x \qquad U = y'$

$V' = -2 \qquad U' = y''$

$V'' = 0 \qquad U^n = y^{n+1}$

$W_2^n = U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V'' + \cdots$

$W_2^n = y^{n+1}(-2x) + n y^n(-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2xy^{n+1} - 2ny^n$

$W_3^n = -y'$

$U^0 = y' \qquad V = -1$

$U' = y'' \qquad V' = 0$

$U^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y$

$U = y \qquad V = 2$

$U' = y' \qquad V' = 0$

$U^n = y^n$

$W_4^n = -2y^n$

Adding all results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1})$
$- 2y^n = 0$

$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

$2\ [e^{x^2}$ FX

$x^3 e^{4x}$

$U = e^{4x} \qquad V = x^3$

$U' = 4e^{4x} \qquad V' = 3x^2$

$U'' = 16e^{4x} \qquad V'' = 6x$

$U^n = 4^n e^{4x} \qquad V''' = 6$

$V^{iv} = 0$

$= (uv)^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!}$

$4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2) \times 4^{n-3} e^{4}}{6}$

$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$4^{n-3} e^{4x} [4^3 x^3 + n4^2 \cdot 3x^2 + n(n-1) 4 \times 3x + n(n-1)(n-2)]$

$= 4^{n-3} e^{4x} [64x^3 + n48x^2 + n(n-1) 12x + n(n-1)(n-2)]$

$= 4^{5-3} e^{4x} [64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)]$

$16e^{4x} [64x^3 + 240x^2 + 240x + 60]$

$y^5 = [1024e^{4x} x^3 + 3840 x^2 e^{4x} + 3840e^{4x} + 960 e^{4x}]$

$x^2 y'' + xy' - y = 0$

$x^2 y^{n+2} + (2n+1) x y^{(n+1)} + (n+1) y^n = 0$

$x^2 y'' + xy' + y = 0$

$W_1 = x^2 y''$

$v = x^2 \quad v' = 2x \quad v^2 = 2$

$u = y^{(2)} \quad u' = y^{(3)} \quad u^2 = y^4$

$u^n = y^{(n+2)}$

$W_1^{(n)} = \sum_{r=0}^{n} {}^nC_r \, u^{(n-r)} v$

$= u^n v^{(0)} + n u^{(n+1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v^2$

$W^{(n)} = y^{(n+2)} x^2 + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n 2$

$= x^2 y^{n+2} + 2xn y^{(n+1)} + n(n-1) y^n$

$W_2 = xy'$

$v = x \quad v' = 1$

$u = y' \quad u' = y^{(2)}$

$$u^{(n)} = y^{(n+1)}$$

$$W_2 = u^n v + n u^{(n-1)} v'$$

$$= y^{(n+1)} x + n y^{(n)}$$

$$W_2^{(n)} = x y^{(n+1)} + n y^{(n)}$$

$$W_3 = y$$

$$u = y \qquad v = 1$$

$$u^n = y^n$$

$$W_3 = u^n v = y^n \quad 1 = y^n$$

$$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2n x y^{(n+1)} + (n)(n-1) y^n + x y^{(n-1)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$$

$$x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + (n^2+1) y^n = 0$$

$$x^2 y(n+2) + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$