We have;

$= (1024x^3 + 13840x^2 + 38\cancel{4}0x + 960)e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$.

2ii $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^2 = 0$

$x^2y'' + xy' + y = 0$

$\cancel{y}$ $u = y^2$ $\qquad V = x^2$

$u^n = y^{(n+2)}$ $\qquad V' = 2x$

$u^{n-1} = y^{(n+1)}$ $\qquad V'' = 2$

$u^{(n-2)} = y^n$ $\qquad V''' = 0$

$y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n(2)}{2!}$

for (xy')

$V = x$ $\qquad u = y$

$V' = 1$ $\qquad u^n = y^{(n+1)}$

$V'' = 0$ $\qquad u^{n-1} = y^n$

Applying

$y^n = u^nV + nu^{(n-1)}V' + \frac{n(n-1)u^{(n-2)}}{2!}V^2 + \dots$

Since $V^2 = 0$

$y^n = y^{(n+1)} \cdot x + n^n$

$y^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + ny^{(n+1)} + ny^n + y$

then.

$y^n = \cancel{x}^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2+1) = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$