Differentiating $y^{(1)}(2x+1)$

Let $v = 2x+1$ $\quad u = y'$

$v' = 2$ $\quad u^{n} = y^{(n+1)}$

$v'' = 0$ $\quad u^{(n-1)} = y^{n}$

Recall from Leibnitz theorem

$$u^{n}v + nu^{(n-1)}v' + \frac{n(n-1)u^{(n-2)}v^{2}}{2!} \qquad 0 = 0$$

$$= y^{(n+1)} \cdot (2x+1) + n(y^{n}) \cdot 2$$

Differentiating $y^{(2)}$ we have ; $y^{(n+2)}$

$$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + 2ny^{n} + 2y^{n}$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{n}(n+1) \Rightarrow \text{proved}$$

2. Using Leibnitz theorem given that $y = x^{3}e^{4x}$ determine $y^{(5)}$

$u = e^{4x}$ $\quad V = x^{3}$

$u^{n} = 4^{n}e^{4x}$ $\quad V' = 3x^{2}$

$u^{(n-1)} = 4^{(n-1)}e^{4x}$ $\quad V'' = 6x$

$u^{(n-2)} = 4^{(n-2)}e^{4x}$ $\quad V''' = 6$

$u^{(n-3)} = 4^{(n-3)}e^{4x}$ $\quad V^{iv} = 0$

where $n = 5$

$4^{5}e^{4x} = 1024e^{4x}$

$u^{(n-1)} = 4^{5-1}e^{4x} = 256e^{4x}$

$u^{(n-2)} = 4^{5-2}e^{4x} = 64e^{4x}$

$u^{n-3} = 4^{5-3}e^{4x} = 16e^{4x}$

$$y^{5} = (1024e^{4x} \cdot x^{3}) + n(256e^{4x} \cdot 3x^{2}) + \frac{n(n-1)64e^{4x} \cdot 6x}{2!}$$

$$+ \frac{n(n-1)(n-2)(16e^{4x}) \cdot 6}{3!}$$