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CHEMICAL ENGINEERING
ENG381

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

where $u = 2x+1$

$v = e^{x^2+x}$

$dy/dx = 2$ $\quad dy/dx = (2x+1)e^{x^2+x}$

$y'' = u\,dv/dx + v\,du/dx$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

Leibnitz theorem: $y'' - y'(2x+1) - 2y = 0$

$y_{11} = 1 \quad v = 1 \quad u^n = y^{n+2}$

$u = y_{11} \quad v' = 0 \quad w_1^n = (2u^n - 0v) = y^{n+2}$

$w_2 = y'(2x+1) \qquad v = 2x+1$

$u = y' \qquad v' = 2$

$u^n = y^{n+1} \qquad v^n = {}^nC_0 u^{n-0} v) + {}^nC_1 u^n v')$

$v'' = 0$

$= u^n v + n v^{n-1}$

$= y^{n+1}(2x+1) n y^n 2$

$W_3 = 2y$

$u = y$

$u^n = y^n \qquad v = 2, v' = 0$

$w_3^n = {}^nC_0 u^n - 0v)$

$= 2y^n$

$y^n - y'(2x+1) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

2 $y = x^3 e^{4x}$

$v^0 = x^3,\ v' = 3x^2,\ v'' = 6x \quad v''' = 6$

$u = e^{4x},\ u' = 4e^{4x},\ u'' = 16e^{4x},\ u''' = 64\ e^{4x}$

$u^n = 4^n e^{4x}$

$y^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v'' + {}^nC_3 u^{n-3}\ v'''$

$= u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!}$

$v''' = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x}\ \frac{3x^2 + (n(n-1) 4^{n-1} e^{4x} + (n-1)(n-2)}{2}$

$\frac{4^{n-3} e^{4x} . 64^0}{3\times 2}$

$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x}\ 3x^2 + n(n-1) 4^{n-2} e^{4x} . 3x + n(n-1)(n-2)$

$4^{n-3} e^{4x}$

$= 4^{n-3} e^{4x} [4^3 x^3 + n4^4\ \ 3x^2 + n(n-1) 4 . 3x + n(n-1)(n-2)]$

$= 4^{n-3}\ e^{4x} [64x^3 + n4\ 8x^2 + 12(n-1) + n(n-1)(n-2)]$

$y^3 = 4^{5-3}\ e^{4x} [64x^3 + (5\times 48) x^2 + 12\times 5(5-1) x + 5(5-1)(5-2)$

$y^3 = 16e^{4x} [64x^3 + 240x^2 + 240x + 60]$

4 $x^2 \frac{d^2y}{dx^2} + \frac{x\,dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w' = x^2 y''$

$u^0 = v = y''$

$v' = y''' u'' = y''''$

$u^n = y^{n+2}$

$w_2 = xy'$ $\qquad v = x,$

$u^0 = u = y'$ $\qquad v' = 1$

$u' = y''$ $\qquad u'' = y'''$ $\qquad v'' = 0$

$u' = y''$

$u^n = y^{n+1}$

$w_1 = y$

$u = y$ $\quad v = 1, v' = 0$

$u' = y', u'' = y''$

$u^n = y^n$

$w_1^n = 1^n C_0 u^{n-0} v^0 + {}^nC_1 v^{n-1} v' + {}^nC_2 u^{n-2} v''$

$= u^n y + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

$= \cancel{y^{n+2} v + n y^{n+1}}$

$= y^{n+2} v + n y^{n+1} \frac{2x + n(n-1)}{x} y^n x$

$= y^{n+2} v + n y^{n+1} 2x + (n-1)n y^n$

$= y^n (y^n x^2 + n y^2 + n(n-1))$

$w_2^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v'' + {}^nC_3 u^{n-3} v'''$

$= u^n v^0 - 1 n u^{n-1} v' + \frac{(n-1)n}{2!} u^{n-2} \cdot 0$

$= y^{n+1} x + n y^{n-1} + 0$

$w_3^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^n v^0 + 0$

$= y_n$

$x^2 y'' + x y' + y = 0$

$w_1 + w_2 + w_3 = 0$

$y^n (y^2 x^2 + n 2 x y + n(n-1)) + y^n (x y + n) + y^n = 0$

at $x = 0$

$y^n n(n-1) + y^n n + y^n = 0$

$n(n-1) + n + 1 = 0$

$y^n \ldots y^n - (n-1) - n y^n$

at $n = 1$

$y = -0 - y'$

$y = -y'$

$= x^2 y^{n+2} + n 2 x y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$

$= x^2 n^{+2} + x y^{n+1} (2n+1) y^n (n^2 - n + n + 1) = 0$

$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$