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16/ENG05/017
Mechatronics
Engineering Mathematics

1. $y = e^{x^2+x}$

Let $x^2+x = u$

$y = e^u$; $\frac{dy}{du} = e^u$

$u = x^2+x$ $\qquad \frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (2x+1)e^u$

$\therefore (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = 2x+1$ $\qquad \frac{du}{dx} = 2$

$v = e^{x^2+x}$ $\qquad \frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$\therefore y'' = (2x+1)y' + 2y$

ii

From y''

$$y^{(n)} = (2x+1)y^{(n-1)} + 2(n-1)y^{(n-2)}$$

$$\therefore y^{(n+2)} = (2x+1)y^{(n+2-1)} + 2(n+2-1)y^{(n-2+2)}$$

$$= (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$$

2 i) $y = x^3 e^{4x}$

$$\sum_{r=0}^{n} {}^nC_r U^{(n-r)} V^{(r)}$$

$V = x^3$; $V^{(1)} = 3x^2$; $V^{(2)} = 6x^1$, $V^{(3)} = 6$, $V^{(4)} = 0$

$U = e^{4x}$; $U^{(1)} = 4e^{4x}$, $U^{(2)} = 16e^{4x}$, $U^{(3)} = 64e^{4x}$, $U^{(4)} = 256e^{4x}$ U

$$y^{(5)} = U^{(5)}V + 5U^{(5-1)}V^{(1)} + {}^5C_2 U^{(5-2)}V^{(2)} + {}^5C_3 U^{(5-3)}V^{(3)} + {}^5C_4 U^{(5-4)}V^{(4)}$$

$$= U^{(5)}V + 5U^{(4)}V^{(1)} + {}^5C_2 U^{(3)}V^{(2)} + {}^5C_3 U^{(2)}V^{(3)} + {}^5C_4 U^{(1)}V^{(4)}$$

$$= 1024e^{4x}x^3 + 5(256e^{4x})(3x^2) + 10(64e^{4x})6x^1 + 10(16e^{4x})6 + 5(4e^{4x})0$$

$$= 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x} + 0$$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$$x^2y'' + xy' + y = 0$$

$\cancel{x^2y''} = {}^nC$

$\cancel{y^{(n)} = \sum_{r=0}^{n} {}^nC_r U^{(n-r)} V^{(r)}}$

ii For $x^2y'' = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^{(n)}2 + 0 - -$

For xy', Let $u = y'$ and $v = x$

$x'y = y^{(n+1)}x + ny^{(n)} + 0 + - - -$

For y, $y = y^{(n)}$

$x^2y'' + xy' + y = 0$

$x^2y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)}$

Collecting like terms

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n+n+1)y^{(n)} = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$\therefore [x^2y'' + xy' + y] = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$