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16/ENG02/040
Computer Engineering.
ENG 381
Assignment II.

(A.) $y = e^{x} + x^2$

Using the chain Rule / function of a function

$y = e^{U}$ $\qquad$ $y = e^{U}$

$U = x + x^2$ $\qquad$ $dy/du = e^{U}$

$\frac{dy}{dx} = 1 + 2x$

$dy/du \times du/dx = (2x+1)e^{U}$

$dy/dx = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x+x^2})$

$y'' = \left(\frac{d^2y}{dx^2}\right) = [2x+1][(2x+1)e^{x^2+x}] + 2[e^{x^2+x}$

$y'' = (2x+1)[(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$\therefore y'' = y'(2x+1) + 2(y)$

(b.) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem.

$W_1^{n} = y''$ $\qquad$ $V_1 = 1$ $\qquad$ $U^{0} = y''$

$V_1 = 0$ $\qquad$ $U^{n} = y^{n+2}$

$W_1^{n} = U^{n}V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \ldots$

$W_1^{n} = y^{n+2}(1) + ny^{n-1}(0) + \ldots$

$\therefore W_1^{n} = y^{n+2}$

$(2x+1)$

$W_2^n = 2xy'$

$v^{(0)} = -2x \qquad u = y'$

$v' = -2 \qquad u' = y''$

$v'' = 0 \qquad u^n = y^{n+1}$

$W_2^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots$

$W_2^n = y^{n+1}(-2x) + n y^n (-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2xy^{n+1} - 2ny^n$

$W_3^n = -y'$

$u^0 = y' \qquad v = -1$

$u' = y'' \qquad v' = 0$

$u^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y$

$u = y \qquad v = 2$

$u' = y' \qquad v' = 0$

$u^n = y^n$

$W_4^n = -2y^n$

Adding all results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} = -2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

$x^3 e^{4x}$

$u = e^{4x} \qquad v = x^3$

$u' = 4e^{4x} \qquad v' = 3x^2$

$u'' = 16e^{4x} \qquad v'' = 6x$

$v''' = 6$

$$= W^{n} = u^{n} v + n u^{n-1} v^{1} + \frac{n(n-1)}{2!} u^{n-2} v^{2} + \frac{n(n-1)(n-2)}{3!}$$

$$4^{n} e^{4x} x^{3} + n 4^{n-1} e^{4x} 3x^{2} + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4}$$

$$= 4^{n} e^{4x} x^{3} + n 4^{n-1} e^{4x} 3x^{2} + n(n-1) 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$= 4^{n-3} e^{4x} [4^{3} x^{3} + n 4^{2} + 3x^{2} + n(n-1) 4 \times 3x + n(n-1)(n-2)]$$

$$= 4^{n-3} e^{4x} [64x^{3} + n 48x^{2} + n(n-1) 12x + n(n-1)(n-2)]$$

$$= 4^{5-3} e^{4x} [64x^{3} + 5(48)x^{2} + 5(4) 12x + 5(4)(3)]$$

$$16 e^{4x} [64x^{3} + 240x^{2} + 240x + 60]$$

$$y^{5} = [1024 e^{4x} x^{3} + 3840 x^{2} e^{4x} + 3840 e^{4} + 960 e^{4x}]$$

If $x^{2} y'' + x y' - y = 0$

$$x^{2} y^{n+2} + (2n+1) x y^{(n+1)} + (n+1) y^{n} = 0$$

$$x^{2} y'' + x y' + y = 0$$

$$W_{1} = x^{2} y''$$

$v = x^{2}$ $\quad v' = 2x$ $\quad v^{2} = 2$

$u = y^{(2)}$ $\quad u' = y^{(3)}$ $\quad u^{2} = y^{4}$

$$u^{n} = y^{(n+3)}$$

$$W_{1}^{(n)} = \sum_{r=0}^{n} {}^{n}C_{r}\, u^{(n-r)} v$$

$$= u^{n} v^{(0)} + n u^{(n+1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v^{2}$$

$$W^{(n)} = y^{(n+2)} x^{2} + n y^{n+1} 2x + \frac{n(n-1)}{2} y^{n} 2$$

$$= x^{2} y^{n+2} + 2xn y^{(n+1)} + n(n-1) y^{n}$$

$$u^{(n)} = y^{(n+1)}$$

$$W_2 = u^n v + n u^{(n-1)} v'$$

$$= y^{(n+1)} x + n y^{(n)}$$

$$W_2^{(n)} = x y^{(n+1)} + n y^{(n)}$$

$$W_3 = y$$

$$u = y \qquad v = 1$$

$$u^n = y^n$$

$$W_3 = u^n v = y^n \quad 1 = y^n$$

$$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2n x y^{(n+1)} + (n)(n-1) y^n + x y^{(n-1)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$$

$$x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + (n^2+1) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$

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