Nnanah Ifeoma Olive
Computer Engineering
16/ENG02/033

18/11/18

1. If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$
and hence prove that
$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^{n}$

Solution

let $u = x^2 + x$ and $y = e^u$

$\frac{du}{dx} = 2x+1$, $\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$

$= e^u(2x+1)$

$= e^{(x^2+x)}(2x+1)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(e^{x^2+x}(2x+1)\right)$

let $b = e^{x^2+x}$ and $a = 2x+1$

$\frac{da}{dx} = 2$

$\frac{db}{dx} = \frac{d}{dx}\left(e^{x^2+x}\right)$

let $c = x^2+x$ and $e = e^u$

$\frac{dc}{dx} = 2x+1$, $\frac{de}{du} = e^u$

$\frac{db}{dx} = e^u(2x+1) = e^{x^2+x}(2x+1)$

$\frac{d^2y}{dx^2} = a\frac{db}{dx} + \frac{da}{dx}$

$= (2x+1)\left(e^{x^2+x}(2x+1) + e^{x^2+x} \cdot 2\right)$

$= \frac{dy}{dx}(2x+1) + 2y$

$\therefore \quad y'' = y'(2x+1) + 2y$ — proven proof that, $\overset{w_1}{\overline{y'(2x+1)}} + \overset{w_2}{\overline{2y}}$ is equal to $y^{(n+2)} = (2x+1)$

$y'' = y'(2x+1) + 2y$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Using leibnitz theorem

for $w_1 = y'(2x+1)$

let $u = y'$, $v = 2x+1$

$u^{(n)} = y^{(n+1)}$, $v^{(1)} = 2$

$u^{(n-1)} = y^{(n)}$, $v^{(2)} = 0$

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}}{2!}v^{2} + \dots$$

$$w_1 = y^{(n+1)}.(2x+1) + ny^{(n)}.2 + \cancel{\frac{n(n-1)u^{(n-2)}}{2!}}.0$$

$$w_1 = y^{(n+1)}(2x+1) + 2ny^{(n)}$$

for $w_2 = 2y$

let $u = 2y$

$u^{(n)} = 2y^{(n)}$

$\therefore y'' = w_1 + w_2$

$= y^{(n+1)}(2x+1) + 2ny^{(n)} + 2y^{(n)}$

$y'' = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

2. Using the leibnitz theorem

given that

1. ~~$y = x^{3}4$~~ $y = x^{3}e^{4x}$

solution

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}}{2!} + \dots$$

let

$u = e^{4x}$, $v = x^{3}$

$u^{(n)} = 4^{(n)}e^{4x}$, $v^{(1)} = 3x^{2}$

$u^{(n-1)} = 4^{(n-1)}e^{4x}$, $v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)}e^{4x}$, $v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)}e^{4x}$, $v^{(4)} = 0$

$$y^{(n)} = 4^{(n)} e^{4x} \cdot x^3 + n 4^{(n-1)} e^{4x} \cdot 3x^2 + \frac{n(n-1) 4^{(n-2)}}{2!} e^{4x} \cdot 6x$$

$$+ \frac{n(n-1)(n-2) 4}{3!}^{(n-3)} e^{4x} \cdot 6 + \frac{n(n-1)(n-2)(n-3) 4^{(n-4)}}{4!} e^{4x} \cdot 0$$

$$= 4^{(5)} e^{4x} x^3 + 5(4^{(5-1)}) e^{4x} 3x^2 + \frac{5(5-1) 4^{(5-2)}}{1 \times 2} e^{4x} 6x$$

$$+ \frac{5(5-1)(5-2) 4}{1 \times 2 \times 3}^{(5-3)} e^{4x} \cdot 6 + \frac{5(5-1)(5-2)(5-3) 4^{(5-4)}}{1 \times 2 \times 3 \times 4} e^{4x} \cdot 0$$

$$= 4^5 e^{4x} x^3 + (5)(4^4) e^{4x} 3x^2 + \frac{(5)\cancel{(4)}(4^3)}{\cancel{2}} e^{4x} \cdot 6x$$

$$+ \frac{5(4)(3) 4^2}{\cancel{6}} e^{4x} \cdot \cancel{6} + 0$$

$$= 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$= e^{4x} (1024 x^3 + 3840 x^2 + 3840 x + 960)$$

(11) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

Show that

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$

solution

$$\underbrace{x^2 y''}_{w_1} + \underbrace{x y'}_{w_2} + \underbrace{y}_{w_3} = 0$$

$w_1 = x^2 y''$

let $u = y''$, $v = x^2$

$u^{(n)} = y^{(n+2)}$, $v' = 2x$

$u^{(n-1)} = y^{(n+1)}$, $v'' = 2$

$u^{(n-2)} = y^{(n)}$, $v''' = 0$

using Leibnitz theorem

$$y^{(n)} = u^{(n)}v + (n)u^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{2} + \cdots$$

$\therefore w_1 + w_2 + w_3 = 0$

$$w_1 = y^{(n+2)}x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}y^{(n)}\cdot 2 + \cancel{\frac{n(n-1)(n-2)}{3!}y^{(n-1)}\cdot 0} \to 0$$

$$w_2 = y^{(n+1)}x + ny^{(n)}\cdot 1 + \cancel{\frac{n(n-1)}{2!}y^{(n+1)}\cdot 0} \to 0$$

$$w_3 = y^{(n)}$$

$$\Rightarrow x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} = 0$$

$$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2 - \cancel{n} + \cancel{n} + 1) = 0$$

$$\therefore x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2+1) = 0 \text{ - proven}$$

Scanned by CamScanner