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Matric: 17/Eng06/091

1. If $y = e^{x^2+x}$

show that

$y'' = y'(2x+1) + 2y$ and hence prove that

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)\, y^n$.

and hence prove that

$y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)\, y^n$

To show that $y'' = y'(2x+1) + 2y$

solve y' for $y = e^{x^2+x}$ --- (1)

$$y' = \frac{d}{dx}\left[e^{x^2+x}\right] = \frac{dy}{dx}$$

Let $x^2 + x = u$

$$\frac{du}{dx} = 2x + 1$$

$$y = e^u$$

$$\frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$\frac{dy}{dx} = e^u \cdot 2x+1 = e^{x^2+x}(2x+1)$$

$$y' = e^{x^2+x}(2x+1) \quad ---- (2)$$

Solving for y''

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^{x^2+x}(2x+1)\right]$$

Solve using product rule

$$\frac{dy}{dx} = \frac{du}{dx} \cdot v + \frac{dv}{dx} \cdot u$$

Let $u = e^{x^2+x}$; $v = 2x+1$

$\frac{du}{dx} = \frac{du}{da} \times \frac{da}{dx}$

when

$a = x^2+x$

$\frac{da}{dx} = 2x+1$

$u = e^a$

$\frac{du}{da} = e^a$; $\frac{du}{dx} = e^a \cdot 2x+1$

subst. $a = x^2+x$

$\frac{du}{dx} = e^{x^2+x} \cdot 2x+1$

$\frac{dv}{dx} = 2$

$\therefore y'' = \frac{du}{dx} \cdot v + \frac{dv}{dx} \cdot u$

$y'' = e^{x^2+x}(2x+1)\cdot(2x+1) + e^{x^2+x} \cdot 2$

$y'' = e^{x^2+x}(2x+1)\cdot(2x+1) + 2e^{x^2+x}$ ----- (3)

subst y in eqn (1) and y'' in eqn (2) into eqn (3)

$y'' = y'(2x+1) + 2y$

from recalling $y = e^{x^2+x}$ and $y' = e^{x^2+x}(2x+1)$

$\therefore y'' = y'(2x+1) + 2y$

Proving $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

from $y'' = y'(2x+1) + 2y$

$y^2 = y'(2x+1) + 2y$

Using Leibnitz theorem

differentiate each term n times

$\underset{N_1}{y^{(2)}} = \underset{N_2}{y^{(1)}(2x+1)} + \underset{N_3}{2y}$

$N_1 = y^{(2)} = y^{(2)} \cdot 1$

$u = y^{(2)}$, $v = 1$

$u^{(n)} = y^{(n+2)}$, $v' = 0$

If $N_2 = y^{(1)} \cdot (2x+1)$

$u = y^{(1)}$ $\quad v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $\quad v' = 2$

$u^{(n-1)} = y^{n}$ $\quad v^2 = 0$

If $N_3 = 2y$

$u = y$ $\quad v = 2$

$u^{(n)} = y^n$ $\quad v' = 0$

$$\therefore y^{(n)} = u^n v + n u^{n-1} v' + \frac{n(n-1) u^{n-2} v^2}{2!} + \cdots$$

$N_1 = N_2 + N_3$

$$y^{(n+2)} = y^{(n+1)} \cdot 2x+1 + n y^{n} \cdot 2 + y^{n} \cdot 2$$

$$y^{(n+2)} = y^{(n+1)} (2x+1) + 2n y^{(n)} + 2y^{(n)}$$

$$y^{(n+2)} = (2x+1) y^{(n+1)} + (2n+2) y^{(n)}$$

$$\underline{y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}}$$

2. Using the Leibnitz theorem, given that

i.) $y = x^3 e^{4x}$, determine $y^{(5)}$.

ii.) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ show that

$$x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^{(n)} = 0.$$

Solution

i.) $u = e^{4x}$; $v = x^3$

$u' = 4e^{4x}$; $v' = 3x^2$

$u'' = 16e^{4x}$; $v'' = 6x$

$u''' = 64e^{4x}$; $v''' = 6$

$u^{iv} = 256e^{4x}$; $v^{iv} = 0$

$u^{v} = 1024e^{4x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^{n-4} v^{iv}$$

$$y^5 = 1024e^{4x} \cdot x^3 + 5(256e^{4x} \cdot 3x^2) + \frac{5(4)}{2!}(64e^{4x} \cdot 6x) + \frac{5(4)(3)}{3!}(16e^{4x} \cdot 6) + 0$$

$$y^5 = x^3 1024e^{4x} + x^2 3840e^{4x} + x\,3840e^{4x} + 960e^{4x}$$

$$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$$

$$y^5 = 64e^{4x}[16x^3 + 60x^2 + 60x + 15]$$

2 ii) $x^2 y'' + xy' + y = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$+ \ldots\ldots\ldots$

$y = uv$

$A = x^2 y''$

$u = y''$; $v = x^2$

$u' = y''^{+1} = y'''$; $v' = 2x$

$u^n = y^{n+2}$; $v'' = 2$

$v''' = 0$

$$y^n = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1) y^n \cdot \cancel{2}}{\cancel{2!}} + 0$$

$$y^n = y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n$$

$B = x y'$

$u = y'$; $v = x$

$u^n = y^{n+1}$; $v' = 1$

$v'' = 0$

$$y^n = y^{(n+1)} x + n y^n$$

$C = y$

$y^n = y^n$

$$x^2 y^{(n+2)} + 2x n y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n =$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 \cancel{- n + n} + 1) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$

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