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$$y = e^{x^2+x}$$
$$y' = (2x+1)e^{x^2+x}$$
$u = 2x+1$; $v = e^{x^2+x}$
$u' = 2$; $v' = (2x+1)e^{x^2+x}$

$$y'' = uv' + vu'$$
$$y'' = (2x+1)\left[(2x+1)e^{x^2+x}\right] + e^{x^2+x}.2$$
but since $y' = (2x+1)e^{x^2+x}$ & $y = e^{x^2+x}$

$\therefore$

$$y'' = y'(2x+1) + 2y \quad \text{— SHOWN}$$
then,
$$\boxed{y'' - y'(2x+1) - 2y = 0}$$
Now, let $A = y''$
$u = y''$; $v = 1$
$u^n = y^{n+2}$; $v' = 0$
$$y^n = u^n v + nv^{n-1}v'$$
$$y^n = (y^{n+2} \times 1) + (ny^{n+1} \times 0)$$
$$y^n = y^{n+2} \implies A'$$

let $B = y'(2x+1)$
$u = y'$; $v = 2x+1$
$u^n = y^{n+1}$; $v' = 2$; $v'' = 0$
$$y^n = y^{n+1}(2x+1) + ny^n.2$$
$$y^n = y^{n+1}(2x+1) + 2ny^n \implies B'$$

let $C = 2y$
$u = y$; $v = 2$
$u^n = y^n$; $v' = 0$
$$y^n = 2y^n + (ny^{n-1} \times 0)$$

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$$y^n = 2y^n \implies C'$$

$\therefore$ Using, $A + B + C = 0$

$$A' - B' - C' = 0$$

$$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$$

$$y^{n+2} = y^{n+1}(2x+1) + y^n(2n+2)$$

$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n \quad \text{— PROVEN.}$$

Question 2

(i)

$$u = e^{4x} \; ; \quad v = x^3$$

$$u^n = 4^n e^{4x} \; ; \quad v' = 3x^2, \; v'' = 6x \; ; \; v''' = 6, \; v^{iv} = 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)4^{n-2}e^{4x}(6x)}{2!} + \frac{n(n-1)(n-2) \times 4^{n-3} \cdot e^{4x} \cdot 6}{3!} + 0$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 \cdot n \cdot 4^{n-1} e^{4x} + 3x(n^2-n)4^{n-2}e^{4x} + n(n^2-3n+2)4^{n-3}e^{4x}$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 \cdot 5 \cdot 4^4 \cdot e^{4x} + 3x(25-5)4^3 e^{4x} + 5(25-15+2) \times 4^2 \cdot e^{4x}$$

$$y^5 = 1024 e^{4x} x^3 + 3x^2 \cdot 5 \cdot 256 \cdot e^{4x} + 3x(20) \cdot 64 \cdot e^{4x} + 5(12) \cdot 16 \cdot e^{4x}$$

$$y^5 = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^5 = 64 e^{4x} x^3 \left(16 + \frac{60}{x} + \frac{60}{x^2} + \frac{15}{x^3}\right)$$

(ii)

$$\boxed{x^2 y'' + xy' + y = 0}$$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3} \cdot v'''}{3!} + \dots$$

$$y = uv$$

Let $A = x^2 y''$

$$u = y'' \; ; \quad v = x^2$$

$$u^n = y^{n+2} \; ; \quad v' = 2x \; ; \; v'' = 2 \; ; \; v''' = 0$$

$$y^n = y^{n+2}x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)y^{n} \cdot 2}{2!} + \frac{n(n-1)(n-2)y^{n-1}}{3!} \times 0$$

$$y^n = y^{n+2}x^2 + ny^{n+1} \cdot 2x + n(n-1)y^n \implies A'$$

Let $B = xy'$

$u = y'$; $v = x$

$u^n = y^{n+1}$; $v' = 1$; $v'' = 0$

$y^n = y^{n+1} \cdot x + ny^n \cdot 1 + \frac{n(n-1)}{2} \times y^{n-1} \times 0$

$y^n = y^{n+1} \cdot x + ny^n \implies B'$

Let $C = y$

$y^n = y^n \implies C'$

Using,

$A + B + C = 0$

$A' + B' + C' = 0$

$y^{n+2}x^2 + ny^{n+1} \cdot 2x + n(n-1)y^n + y^{n+1} \cdot x + ny^n + y^n = 0$

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2 - n + n + 1) = 0$

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2+1) = 0$ - PROVEN.

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