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Mechanical Engineering
Eng 381

1 If $y = e^{x^2+x}$
Show that
$y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)\,y^{(n+1)} + 2(n+1)\,y^n$

2 Using the leibniz theorem, given that
i $y = x^3 e^{4x}$, determine y^3
ii $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$,

$x^2 y^{(n+2)} + (2n+1)\,x\,y^{(n+1)} + (n^2+1)\,y^n = 0$

Soln
1 $y = e^{x^2+x}$
$\ln y = x^2 + x$
Differentiating both sides
$\dfrac{1}{y}\dfrac{dy}{dx} = 2x+1$
Multiply both sides by y
$\dfrac{dy}{dx} = (2x+1)\,y$
$\dfrac{d^2y}{dx^2} = U\,{}^{dv}/_{dx} + V\,{}^{du}/_{dx}$
$V = 2x+1$ $\quad {}^{dv}/_{dx} = 2$
$u = y$ $\quad {}^{du}/_{dx} = {}^{du}/_{dy} \cdot {}^{dy}/_{dx} = 1 \cdot {}^{dy}/_{dx}$

So that
$\dfrac{d^2y}{dx^2} = (2x+1)\cdot\dfrac{dy}{dx} + 2y$

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$$y'' = y'(2x+1) + 2y$$

$$y'' = y'(2x+1) + 2y$$
$$y^2 = y'(2x+1) + 2y$$

Differentiating $y'(2x+1)$

Let

$V = 2x+1$ $\qquad$ $u = y'$

$V' = 2$ $\qquad$ $u^n = y^{(n+1)}$

$V^2 = 0$ $\qquad$ $u^{n-1} = y^n$

$$y^{(n+1)} \cdot (2x+1) + n(y^n) \cdot 2$$

Differentiating $2y$ we have; y^{n+2}

$(2y = 2y^n)$

$$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + 2ny^n + 2y^n$$

$$\Rightarrow y^{n+2} = (2x+1)y^{(n+1)} + 2y^n(n+1)$$

24 $\quad y = x^3e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\qquad$ $V = x^3$

$u^n = 4^n e^{4x}$ $\qquad$ $V' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\qquad$ $V^2 = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\qquad$ $V^3 = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\qquad$ $V^4 = 0$

where $n = 5$

$4^5 e^{4x} = 1024e^{4x}$

$u^{(n-1)} = 4^{(5-1)} e^{4x} = 256e^{4x}$

$u^{(n-2)} = 4^{(5-2)} e^{4x} = 64e^{4x}$

$u^{(n-3)} = 4^{(5-3)} e^{4x} = 16e^{4x}$

$y^3 = [1024e^{4x} \cdot x^3] + n[256e^{4x} \cdot 3x^2] + \frac{n(n-1)\, 64e^{4x} \cdot 6x}{2!}$

$+ \frac{n(n-1)(n-2)(16e^{4x})}{3!} \cdot 6$

we have

$[1024x^3 + 3840x^2 + 3840x + 960]e^{4x}$
$=$

$y^{(3)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

2ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$

$x^2 y'' + xy' + y = 0$

$u = y^2 \qquad v = x^2 \quad \Rightarrow \quad \boxed{y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^{n}(2)}{2!}} + v^3 = 0$

$u^n = y^{(n+2)} \qquad v' = 2x$

$u^{n-1} = y^{(n+1)} \qquad v^2 = 2$

$u^{(n-2)} = y^n \qquad v^3 = 0$

For $(xy'$

$v = x \qquad u = y'$

$v' = 1 \qquad u^n = y^{n+1}$

$v^2 = 0 \qquad u^{(n-1)} = y^n$

Applying

$y^n = u^n v + n u^{n-1} v'$

Since $v^2 = 0$

For (xy')

$y^n = y^{(n+1)} \cdot x + ny^{(n)}$

$y^n = x^2 y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n$

$$= x^2 y^{(n+2)} + x\, y^{(n+1)} (2n+1) + y^{n} (n^2+1) = 0$$

$$y^{n} = x^2 y^{(n+2)} + x\, y^{(n+1)} (2n+1) + y^{n} (n^2+1) = 0$$