SAM-OGBONNA CHIDIEBERE.
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CHEMICAL ENGINEERING.
ENG 381 -

(1) If $y = e^{x^2+x}$.
Show that $y'' = y'(2x+1) + 2y$.
and prove that $y^{(n+2)} = (2x+1)y^{(n-1)} + 2(n+1)y^n$.

Soln.

$y' = (2x+1)e^{x^2+x}$

To find y'' we use product rule.

Let $u = 2x+1$, $v = e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$.

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$ $\frac{du}{dx} = 2$.

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$.

Since $y' = (2x+1)e^{x^2+x}$ and $y = e^{x^2+2}$.

$y'' = y'(2x+1) + 2y$.

$y'' - y'(2x+1) - 2y = 0$.

Let $w_1 = y''$.

$u^{(0)} = y''$, $u^{(1)} = y'''$, $u^{(11)} = y^{iv}$.

$u^{(n)} = y^{(n+2)}$, $w_1^{(n)} = y^{(n+2)}$.

Let $w_2 = y'(2x+1)$.

$u^{(0)} = y'$, $u^{(1)} = y''$; $u^{(11)} = y'''$, $u^{(n)} = y^{(n+1)}$.

$v^{(0)} = 2x+1$, $v^{(1)} = 2$, $v^{(11)} = 0$.

$w_2^{(1)} = y^{(n+1)}. 2x+1 + ny^n.2$.

$= y^{n+1}(2x+1) + 2ny^n$.

$w_3 = 2y$.

$u = y$, $u^n = y^n$.

$u^{(n)} = y^{(n)}(2)$.

$w^3 = 2y^n$.

$y^{(n+2)} - (y^{(n+1)}(2x+1) + 2ny^n) - 2y^n = 0$.

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$.

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(2) $y = x^3 e^{4x}$, determine $y^{(5)}$.

Let $V = x^3$, $u = e^{4x}$.

$V^{(0)} = x^3$, $V^{(1)} = 3x^2$, $V^{(11)} = 6x$.

$u^{(0)} = e^{4x}$, $u^{(1)} = 4e^{4x}$, $u^{(11)} = 16e^{4x}$, $u^{(111)} = 64e^{4x}$.

$u^{(n)} = 4^n . e^{4x}$.

$y^{(n)} = u^{(n)} . V^{(0)} + n u^{(n-1)} V^{(1)} + n(n-1) u^{(n-2)} V^{(2)}$.

$y^{(n)} = 4^n . e^{4x} . x^3 + n 4^{n-1} e^{4x} . 3x^2 + \frac{n(n-1)}{2} . 4^{(n-2)} e^{4x} . 6x + \frac{n(n-1)(n-2) 6}{6} . 4^{n-3} e^{4x}$

$y^{(n)} = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} . 3x^2 + n(n-1) . 2^{n-2} e^{4x} . 6x + n(n-1)(n-2)$

$y^{(5)} = 4^{5-3} . e^{4x} [64x^3 + 48nx^2 + 12n(n-1)x + n(n-1)(n-2)] 4^{n-3} e^{4x}$.

$y^{(5)} = 4^2 . e^{4x} [64x^3 + 240x^2 + 240x + 60]$.

$y^{(5)} = 16 . e^{4x} [64x^3 + 240x^2 + 240x + 60]$.

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{x4}$.

(1) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, Show that $x^2 y^{(n-2)} + (2n+1) xy^{(n+1)} + (n^2+1) y^{(n)} = 0$.

$x^2 y'' + xy' + y = 0$.

$w_1^{(n)} = x^2 y''$.

$V^{(0)} = x^2$, $V^{(1)} = 2x$, $V^{(11)} = 2$.

$u^{(0)} = y''$, $u^{(1)} = y'''$, $u^{(11)} = y^{IV}$ $u^{(n)} = y^{(n+2)}$.

$w_1^{(n)} = y^{(n+2)} . x^2 + ny^{n+1} . 2x + n(n-1) y^{(n)} . 2$

$w_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n$

$w_2^{(n)} = xy'$.

$u^0 = y'$, $u^{(1)} = y''$, $u^{(11)} = y'''$; $u^n = y^{(n+1)}$.

$V^0 = x$, $V' = 1$

$w_2^{(n)} = y^{(n+1)} . x + ny^n . 1$.

$w_2^{(n)} = xy^{(n+1)} + ny^n$.

$w_3 = y$.

$u^{(0)} = y$, $u^{(1)} = y'$, $u^n = y^{(n)}$.

$V^{(0)} = 1$, $V^{(1)} = 0$.

$w_3^{(n)} = y^{(n)} . 1 + u^{(n-1)} (0)$.

$w_3^{(n)} = y^n$.

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1) y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1) y^n + ny^n + y^n$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n(n-1) + n + 1) = 0$

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$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 \cancel{+n} + \cancel{n} + 1) = 0.$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0.$

Hence,

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0 = x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$

$\frac{n-2)(n-2)\cancel{6}}{\cancel{6}}$

4.

$n+1)$

0.