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16/ENG02/055
COMPUTER ENG
ENG 381

1. $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

where: $u = 2x+1$, $v = e^{x^2+x}$

$du/dx = 2$ $\quad dv/dx = (2x+1)e^{x^2+x}$

$y'' = u\,dv/dx + v\,du/dx$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}$ ②

$y'' = y'(2x+1) + 2y$

from Leibnitz theorem

$y' - y'(2x+1) - 2y = 0$

$w_1 = y''$ $\quad u = y''$ $\quad v = u$ $\quad u^n = y^{n+2}$

$v' = 0$ $\quad m_1^n = {}^nC_0 u^{n-0} v^{-2} = y^{n+2}$

$w_2 = y'(2x+1)$ $\quad y = y$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$

$v'' = 0$ $\quad v_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^2 v + n u^{n-1}$

$y^{n+1}(2x+1) + n y^{n-2}$

$w_3 = 2y$

$u = y$ $\quad v = 2$ $\quad v' = 0$

$u^n = y^n$

$w_3^n = {}^nC_0 u^{n-0} v$

$= 2y^n$

$y^n - y'(2x+1) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2n y^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

2) $y = x^3 e^{4x}$

$v^0 = x^3,\ v' = 3x^2,\ v'' = 6x,\ v''' = 6$

$u = e^{4x},\ u' = 4e^{4x}$

$u'' = 16e^{4x}\ u''' = 64e^{4x}$

$y^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v + {}^nC_2 u^{n-2} v'' + {}^nC_3 u^{n-3} v'''$

$= u^n v' + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} + \frac{n(n-1)(n-2) v'''}{3!}$

$\frac{4^{n+3} e^{4x} - 6}{3 \times 2}$

$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x +$
$n(n-1)(n-2) 4^{n-3} e^{-4x}$

$= 4^{n-3} e^{4x} [4^3 x^3 + n4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2)]$

$= 4^{n-3} e^{4x} [64x^3 + n\,48x^2 + 12(n-1)x + n(n-1)(n-2)]$

$y^3 = 4^{5-3} e^{4x} [64x^3 + (5 \times 48)x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2)$

$= 16e^{4x} [64x^3 + 240x^2 + 24x + 60]$

II) $\frac{x^2 d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

$w_1 = x^2 y''$

$u^0 = v = y''$

$v' = y''',\ v'' = y''''$

$u^n = y^{n+2}$

$w_0 = x y'$

$u^0 = u = y'$

$u' = y'',\ y'' = y''$ $\quad v = x,\ v' = 1,\ v'' = 0$

$u' = y'',\ u'' = y''$

$u^n = y^{n+1}$

$w_0 = y$

$u = y$

$u = y, v = y^n$

$u^n = y^n$

$W_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 v^{n-1} v' + {}^nC_2 v^{n-2} v''$

$= u^n v + n u^{n-1} v^2 + \frac{n(n-1)}{2!} u^{n-2} v''$

$y^{n+2} v + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n 2$

$= y^{n+2} v + n y^{n+1} 2x + (n-1) n y^n$

$= y^n [y^2 x^2 + n y 2x + n(n-1)]$

$W_0^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_0 u^{n-2} v''$

$= u^n v^0 + n u^{n-1} v' + \frac{(n-1)n}{2!} u^{n-2} = 0$

$y^{n+1} x + n y^n - 1 + 0$

$y^n (xy + n)$

$W_0^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^n y^0 + 0$

y^n

$x^2 y'' + xy' + y = 0$

$W_1 + W_2 + W_3 = 0$

$y^n [y^2 x^2 + n 2xy + n(n-1)] + y^n (xy + n) + y^n = 0$

$x = 0$

$y^n n(n-1) + y^n n + y^n = 0$

$n(n-1) + n + 1 = 0$

$y^n - y^n n(n-1) - n y^n$

$\text{If } n = 1$

$y = -0 - y'$

$y = -y'$

$= x^2 y^{n+1} + n 2x y^{n+1} + n(n-1) y^n + x y^{n+1} n y^n + y^n = 0$

$= x^2 y^{n+2} + x y^{n+1} (2n+1) + y^n (n^2 - n + n + 1) = 0$

$= x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 + 1) y = 0$