

2)

$$y = x^3 e^{4x}$$

$$v^0 = x^3, v^1 = 3x^2, v^2 = 6x, v^3 = 6$$

$$u^0 = e^{4x}, u^1 = 4e^{4x}, u^2 = 16e^{4x}, u^3 = 64e^{4x}$$

$$u^n = 4^n e^{4x}$$

$$y^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v^1 + {}^nC_2 u^{n-2} v^2 + {}^nC_3 u^{n-3} v^3$$

$$= u^n v^0 + n u^{n-1} v^1 + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$$

$$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 4^{n-3} e^{4x} \cdot 6$$

$$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$= 4^{n-3} e^{4x} (4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2))$$

$$= 4^{n-3} e^{4x} (64x^3 + 240x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2))$$

$$y^5 = 4^{5-3} e^{4x} (64x^3 + 240x^2 + 240x + 60)$$

$$y^5 = 16 e^{4x} (64x^3 + 240x^2 + 240x + 60)$$

ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

$$\begin{array}{ccc} x^2 y'' & + & x y' + y = 0 \\ \Downarrow & & \Downarrow \quad \Downarrow \\ w_1 & & w_2 \quad w_3 \end{array}$$

$$w_1 = x^2 y''$$

$$u^0 = y'', u^1 = y'''$$

$$v^0 = x^2, v^1 = 2x$$

$$u^n = y^{n+2}$$

$$u^2 = y^{IV}$$

$$v^2 = 2$$

$$w_2 = x y'$$

$$u^0 = y'$$

$$u^1 = y''$$

$$u^2 = y'''$$

$$v^0 = x$$

$$v^1 = 1$$

$$v^2 = 0$$

$$u^n = y^{n+1}$$

$$w_3 = y$$

$$u^0 = y$$

$$v^0 = 1$$

$$u^1 = y'$$

$$v^1 = 0$$

$$u^n = y^n$$