

1.)

$$y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$u = 2x+1$$

$$du/dx = 2$$

$$y = e^{x^2+x}$$

$$dy/dx = (2x+1)e^{x^2+x}$$

$$y'' = u \frac{dy}{dx} + y \frac{du}{dx}$$

$$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

$$y'' = y'(2x+1) + 2y$$

From Leibnitz Theorem.

$$y'' - y'(2x+1) - 2y = 0$$

$$\downarrow$$

$$w_2$$

$$\downarrow$$

$$w_3$$

$$w_1 = y''$$

$$u^0 = y''$$

$$v^0 = 1$$

$$; u' = y'''$$

$$v' = 0$$

$$u^n = y^{n+2}$$

$$w_1^n = {}^nC_0 u^{n-0} v^0 + \dots$$

$$w_1 = y^{n+2}$$

$$w_2 = y'(2x+1)$$

$$u^0 = y'$$

$$v^0 = 2x+1$$

$$u' = y''$$

$$v' = 2$$

$$w_2^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v^1$$

$$= y^{n+1}(2x+1) + n y^n 2$$

$$w_3 = 2y$$

$$u^0 = y$$

$$v^0 = 2$$

$$u' = y''$$

$$u' = y''$$

$$v' = 0$$

$$w_3 = {}^nC_0 u^{n-0} v$$

$$; w_3^n = 2y^n$$

$$; w_1 + w_2 + w_3$$

$$y^{n+2} - y^{n+1}(2x+1) - 2n y^n - 2y^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$