

$$\begin{aligned}
 W_1^n &= {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V^1 + {}^nC_2 U^{n-2} V^2 \\
 &= U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 \\
 &= y^{n+2} V + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n 2 \\
 &= y^{n+2} V + n y^{n+1} 2x + n(n-1) y^n
 \end{aligned}$$

$$\begin{aligned}
 W_2^n &= {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V^1 + {}^nC_2 U^{n-2} V^2 + {}^nC_3 U^{n-3} V^3 \\
 &= U^n V^0 + n U^{n-1} V^1 + \frac{n(n-1)}{2!} U^{n-2} V^2 + 0 \\
 &= y^{n+1} x + n y^n \cdot 1 + 0
 \end{aligned}$$

$$\begin{aligned}
 W_3^n &= {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V^1 + 0 \\
 &= U^n V^0 + 0 \\
 &= y^n
 \end{aligned}$$

$$W_1 + W_2 + W_3 = 0$$

$$x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n + y^{n+1} + n y^n + y^n = 0$$

$$x^2 y^{n+2} + 2x y^{n+1} (n+1) + y^n (n^2 - n + n + 1) = 0$$

$$x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2 + 1) y^n = 0$$