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17/ENG01/036
CHEMICAL ENGINEERING
ENG 381

1a $y = e^{x+x^2}$

Using the Chain rule

$y = e^u$ $y = e^y$

$u = x + x^2$ $dy/du = e^u$

$du/dx = 1 + 2x$

$dy/du \times du/dx = (2x+1)e^u$

$dy/dx = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x+x^2})$

$y'' = (2x+1)[(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

$= (2x+1)[(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

$(2x+1)(e^{x^2+x}) = y'$

$y'' = y'(2x+1) + 2(y)$

1b $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem

$W_1^n = y''$

$V_1 = 1$ $U^0 = y''$

$V_1' = 0$ $U^n = y^{n+2}$

$W_1^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \ldots$

$W_1^n = y^{n+2}(1) + ny^{n+1}(0) + \ldots$

$W_1^n = y^{n+2}$

Expanding $-y'(2x+1)$

$W_2^n = -2xy'$ $W_3 = -y'$

$W_2^n = -2xy'$

$V^{(0)} = -2x$ $U = y'$

$V^{(1)} = -2$ $U' = y''$

$V^{(2)} = 0$ $U^n = y^{n+1}$

$W_2^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^{(2)} + \ldots$

$W_2^n = y^{n+1}(-2x) + ny^n(-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2xy^{n+1} - 2ny^n$

$W_3^n = -y'$

$U^0 = y' \quad V = -1$

$U' = y'' \quad V' = 0$

$U^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y$

$U = y \quad V = -2$

$U' = y' \quad V' = 0$

$U^n = y^n$

$W_4^n = -2y^n$

Adding all the results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$

$y^{n+2} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2a $y = x^3 e^{4x}$

$U = e^{4x} \quad V = x^3$

$U' = 4e^{4x} \quad V' = 3x^2$

$U'' = 16e^{4x} \quad V'' = 6x$

$U^n = 4^n e^{4x} \quad V''' = 6$

$V^{iv} = 0$

$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!} U^{n-2}V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3}V^3 + \ldots$

$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4x} 6$

$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + n(n-1)4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$y^n = 4^{n-3} e^{4x} [4^3 x^3 + n4^2 \times 3x^2 + n(n-1)4 \times 3x + n(n-1)(n-2)]$

$y^n = 4^{n-3} e^{4x} [64x^3 + n48x^2 + n(n-1)12x + n(n-1)(n-2)]$

$y^5 = 4^{5-3} e^{4x} [64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)]$

$y^5 = 16e^{4x} [64x^3 + 240x^2 + 240x + 60]$

$\therefore \quad y^5 = [1024e^{4x}x^3 + 3840x^2e^{4x} + 3840e^{4x}x + 960e^{4x}]$

2b $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y^{n+2} + (2n+2)xy^{(n+2)} + (n^2+1)y^n = 0$

$x^2y'' + xy' + y = 0$

$\downarrow W_1 \quad \downarrow W_2 \quad \downarrow W_3$

$W_1 = x^2y''$

$V = x^2, \quad V' = 2x \quad V'' = 2$

$U = y^2, \quad U' = y^3, \quad U^2 = y^4$

$W_1^n = \sum_{i=0}^{n} {}^nC_r\, U^{(n-r)} U^r$

$= U^nV^0 + nU^{n+1}V' + \frac{n(n-1)}{2!} U^{(n-2)}V^2$

$= y^{n+2}x^2 + ny^{n+1}2x + \frac{n(n-1)}{\cancel{2}} y^n \cancel{2}$

$= x^2y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n$

$W_2 = xy' \quad V = x \quad V' = 1$

$U = y'. \quad U' = y^2$

$U^n = y^{(n+1)}$

$W_2^{(n)} = U^{(n)}V + nU^{(n-1)}V'$

$= y^{(n+1)}x + ny^n \cdot 1$

$= xy^{n+1} + ny^n$

$W_3 = y$

$U = y, \quad U^n = y^n$

$= U^nV = y^n \cdot 1 = y^n$

$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$

$x^3y^{(n+2)} + 2nxy^{(n+2)} + n(n-1)y^n + xy^{(n+2)} + ny^n + y^n = 0$

$x^2y^{(n+2)} + x(1+2n)(y)^{n+1} + (n(n-1) + n + 1)y^n = 0$

$x^2y^{(n+2)} + x(1+2n)y^{(n+1)} + (n^2-n+1+1)y^n = 0$

$x^2y^{(n+2)} + x(2n+1)y^{(n+1)} + (n^2+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$