$$y^{n+2} - 2xy^{n+1} - 2ny^{n} - y^{n+1} - 2y^{n} = 0$$
$$y^{n+2} = 2xy^{n+1} + 2ny^{n} + y^{n+1} + 2y^{n}$$
$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^{n} \text{ ------- proven}$$

2. $y = x^3 e^{4x}$ $\Rightarrow$ determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u' = 4e^{4x}$ $\quad v' = 3x^2$

$u'' = 16e^{4x}$ $\quad v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad v''' = 6$, $v^{iv} = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3}$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4x} 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + (n-1) n 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^n = 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 \times 3x^2 + n(n-1) 4 \times 3x + n(n-1)(n-2)]$$

$$y^n = 4^{n-3} e^{4x} [64x^3 + n48x^2 + n(n-1)12x + n(n-1)(n-2)]$$

$$y^n = 4^{5-3} e^{4x} [64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)]$$

$$y^5 = 16e^{4x} [64x^3 + 240x^2 + 240x + 60]$$

$$y^5 = [1024e^{4x}x^3 + 3840x^2 e^{4x} + 3840 e^{4x} x + 960 e^{4x}]$$

$= 0$

$w_1^n = y^{n+2}(1) + ny^{n+1}(0) + \ldots$

$w_1^n = y^{n+2}$

Expanding $-y'(2x+1)$

$w_2^n = -2xy'$ $w_3 = -y'$

$w_2^n = -2xy'$

$u^0 = -2xy$ $u = y^2$

$u' = -2$ $u' = y''$

$u^n = 0$ $u^n = y^{n+1}$

$\therefore w_2^n = u^n u + nu^{n-1}u' + \frac{n(n-1)}{2} u^{n-2}u'' + \ldots$

$w_2^n = y^{n+1}(-2x) + ny^n(-2) \frac{n(n-1)}{2} \times y^{n-1}(0)$

$w_2^n = -2xy^{n+1} - 2ny^n$

$w_3^n = -y'$

$u^0 = y'$ $u = -1$

$u' = y''$ $u' = 0$

$u^n = y^{n+1}$

$w_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$w_4^n = 2y$

$u = y$ $v = -2$

$u' = y'$ $v' = 0$

$u^n = y^n$

$w_4^n = 2y^n$

Adding all the results we have

$w_1^n + w_2^n + w_3^n + w_4^n = y^{n+1} - 2xy^{n+1} - 2ny^n - (y^{n+1} - 2y^n = 0$

2

3. If $x^2\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0$. Show that x^2y^{n+2} $+ (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Taking the given equation and differentiating n' times we have:

If $w = x^2y''$ then $w^{(n)}$ will be $y^{n+2}x^2 + ny^{(n+1)}2x$

$w = xy'$ then $w^{(n)} = y^{n+1}x + ny^{(n)}.1 + 0 \ldots$

$w = y$ then $w^n = y^n$

$$\left[x^2\frac{d^2y}{dx^2} + \frac{xdy}{dx} + y\right]^n = x^2y^{(n+2)} + [2n+1]$$

$$xy^{(n+1)} + (n^2+1)y^n = 0$$

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1. $y = e^{x+x^2}$

Using the chain rule / function of a function

$y = e^u$ $\quad$ $y = e^u$

$u = x+x^2$ $\quad$ $\frac{dy}{dx} = e^u$

$\frac{dy}{du} + \frac{dy}{dx} = [2x+1]\, e^u$

$\therefore \frac{dy}{dx} = (2x+1)\, e^{(x+x^2)}$

$\therefore y' = (2x+1)\,(e^{x+x^2})$

$y''\left(\frac{d^2y}{dx^2}\right) = [2x+1][2x+1\,(e^{x^2+x})] + 2[e^{x^2+x}]$

$y'' = (2x+1)[(2x+1)\,e^{x^2+x}] + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$ and $(2x+1)\,e^{x^2+x} = y'$

$y'' = y'(2x+1) + 2(y)$ .... prove

b. $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz's theorem

$u_1^n = y''$ $\quad u_1 = 1$ $\quad u' = y''$

$u_1' = 0$ $\quad u^n = y^{n+2}$