$y'' - y'(2x+1) - 2 = 0$

$y^{n+2} - (y^{n+1}(2x+1) + 2ny^{n}) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^{n} - 2y^{n} = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^{n}(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^{n}(n+1)$

proved

(2) $y = x^3 e^{4x}$ Determine $y^{(5)}$ using Leibnitz theorem

Solution

$y^{(5)} = u^{5}v + 5u^{(5-1)}v^{(1)} + \frac{5(5-1)u^{(5-2)}v^{(2)}}{2!} + \frac{5(5-1)(5-2)u^{(5-3)}v^{(3)}}{3!}$

$+ \frac{5(5-1)(5-2)(5-3)u^{(1)}v^{(4)}}{4!} + uv^{(5)}$

$y^{(5)} = u^{5}v + 5u^{(4)}v^{(1)} + \frac{20u^{(3)}v^{(2)}}{2!} + 60u^{(2)}v^{(3)} + \frac{120u^{(1)}v^{(4)}}{4!}$

$+ uv^{(5)}$

$y^{(5)} = u^{(5)}v + 5u^{(4)}v^{(1)} + 10u^{(3)}v^{(2)} + 10u^{(2)}v^{(3)} + 5u^{(1)}v^{(4)}$

$+ uv^{(5)}$

where, $u = e^{4x}$ $\quad$ $v = x^3$

$u^{(5)} = (4)^5 e^{4x}$ $\quad$ $v' = 3x^2$

$u^{(5)} = 1024e^{4x}$ $\quad$ $v^{(2)} = 6x$

$u^{(4)} = (4)^4 e^{4x} = 256e^{4x}$

$u^{(3)} = (4)^3 e^{4x} = 64e^{4x}$

$u^{(2)} = (4)^2 e^{4x} = 16e^{4x}$

$u^{(1)} = (4)^1 e^{4x} = 4e^{4x}$

$y^{(5)} = 1024e^{4x} \cdot x^3 + 5 \cdot 256 \cdot e^{4x} \cdot 3x^2 + 10 \cdot 64e^{4x} \cdot 6x + 10 \cdot 16e^{4x} \cdot 6 +$
$15 \cdot 16e^{4x} \cdot 0 + e^{4x} \cdot 0$

$y^{(5)} = 64e^{4x}\{16x^3 + 60x^2 + 60x + 15\}$

(11) $x^2 \frac{d^2y}{dx^2} + x\,dy/dx + y = 0$

This equation can be re-written as

$x^2 y'' + xy' + y = 0$

Taking each term and differentiating n times using Leibnitz theorem

If $k_1 = x^2 y''$

Let $u = y^{(2)}$

$u^{(n)} = y^{(n+2)}$

$u^{(n-1)} = y^{(n+1)}$

$u^{(n-2)} = y^{(n)}$

$v = x^2$

$v^{(1)} = 2x$

$v^{(2)} = 2$

$v^{(3)} = 0$

From

$$y^{(n)} = u^{(n)}v + n u^{(n-1)} v^{(1)} + \frac{n(n-1) u^{(n-2)} v^{(2)}}{2!} +$$

$$k_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} \frac{2x + n(n-1) y^{(n)}}{2} 2 + 0$$

$$W_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$$

If $W_2 = xy'$

Let $u = y^{(1)}$

$u^{(n)} = y^{(n+1)}$

$u^{(n-1)} = y^{(n)}$

$v = x$

$v^{(1)} = 1$

$v^{(2)} = 0$

$$W_2^{(n)} = y^{(n+1)} \cdot x + ny^{(n)} \cdot 1 + 0$$

If $W_3 = y$

$u = y$

$u^{(n)} = y^{(n)}$

Therefore $W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$

That is

$$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + y^{(n+1)}x + ny^{(n)} + y^{(n)}$$

$$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2-n+n+1) = 0$$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$

16/Eng/04/053

Elect/Elect

1) If $y = e^{x^2+x}$ show $y'' = y'(2x+1) + 2y$

and hence prove ~~that~~ $y^{(n+1)} = (2x+1)y^{(n-1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$

$y' = (2x+1)\,e^{x^2+x}$

$u = 2x+1$

$\frac{du}{dx} = 2$

$v = e^{x^2+x}$

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\,dv/dx + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}\,(2)$

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

$|a| = y''$

$u = y''$

$u^n = u^{n+2}$

$v = 1$

$= 0$

$W^n = {}^nC_0\,u^{n-0}\,v^0 + {}^nC_1\,u^{n-1}\,v'$

$= y^{n+2} + 0$

$= y^{n+2}$

$W_2 = y'(2x+1)$

$u = y'$

$u^n = y^{n+1}$

$W_2^n = {}^nC_0\,u^{n-0}\,v^0 + {}^nC_1\,u^{n-1}\,v'$

$= u^n v + n u^{n-1} v'$

$= y^{n+1}(2x+1) + 2ny^n$

$W_3 = 2y$

$u = y$

$u' = y$

$u^n = y^n$

$v = 2$

$v' = 0$

$W_3^n = {}^nC_0\,u^{n-0}\,v^0$

$= u^n v$

$W_3 = 2y^n$

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