OGBOMIDA Philip Ofeoshi
Mechanical Engineering
17/ENG06/101
ENG 381
Engineering Mathematics III

(1). If $y = e^{x^2+x}$
Show that:
$y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^u \quad ; \quad u = x^2 + x$

$\frac{dy}{dx} = e^u \quad ; \quad \frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = 2x+1 \cdot e^u$

$y' = 2x+1\,(e^u)$

$y' = 2x+1\,(e^{x^2+x})$

$\frac{d^2y}{dx^2} = 2x+1\left(2x+1\,[e^{x^2+x}]\right) + e^{x^2+x} \cdot 2$

$y'' = 2x+1\,[y'] + 2(y)$

$\boxed{y'' = y'(2x+1) + 2y}$

For $y'' \rightarrow y^{(n+2)}$

$y'(2x+1) \; ; \quad v = 2x+1 \qquad u^n = y^{n+1}$

$v' = 2 \qquad u^{n+1} = y^{n+}$

$2y \rightarrow 2y'$

$y^{(n+2)} = y^{n+1} \cdot 2x+1 + n\left[y^n 2\right] + 2y^n$

$\boxed{y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n}$

(2). $y = x^3 e^{4x}$, determine $y^5$

Solution

where $v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x}$

$v''' = 6$ $\quad u^n = 4^n e^{4x}$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$y^n = 4^n e^{4x} \cdot x^3 + n\left[4^{n-1} e^{4x} \cdot 3x^2\right] + \frac{n(n-1)}{2!}\left[4^{n-2} e^{4x} \cdot 6x\right] + \frac{n(n-1)(n-2)}{3!}\left[4^{n-3} e^{4x} \cdot 6\right]$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n\left[4^{n-1} e^{4x}\right] + n(n-1)3x\left[4^{n-2} e^{4x}\right] + n(n-1)(n-2)\left[4^{n-3} e^{4x}\right]$$

Therefore:

$$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 \cdot 5\left[4^{5-1} e^{4x}\right] + 5(5-1)3x\left[4^{5-2} e^{4x}\right] + 5(5-1)(5-2)\left[4^{5-3} e^{4x}\right]$$

$$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 \cdot 5\left[4^4 e^{4x}\right] + 20 \cdot 3x\left[4^3 e^{4x}\right] + 60\left[4^2 e^{4x}\right]$$

$$\boxed{y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}}$$

(3). If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; show that:

$$x^2 y^{n+2} + (2n+1)x y^{(n+1)} + (n^2+1) y^n = 0$$

Solution

$x^2 y'' \rightarrow u^n = y^{n+2}$ $\quad v = x^2$

$v' = 2x$

$v'' = 2$

$$y_1'' = y^{n+2} \cdot x^2 + n y^{n+1} 2x + n(n-1) y^n \cdot 2 / 2!$$

$x y' \rightarrow u^n = y^{n+1}$ $\quad v = x$

$v' = 1$

$$y_1' = y^{n+1} \cdot x + n y^n \cdot 1$$

$y \rightarrow y^n$

$$y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + n(n-1) y^n + y^{n+1} \cdot x + n y^n \cdot 1 + y^n = 0$$

$$y^{n+2} \cdot x^2 + x(2n+1) y^{n+1} + (n^2-n) y^n + n y^n + y^n = 0$$

$$y^{n+2} \cdot x^2 + (2n+1) x y^{(n+1)} + y^n \left( \cancel{4}\left[ n^2 - n \right] + \cancel{4}n + 1 \right) = 0$$

$$y^{n+2} \cdot x^2 + (2n+1) x y^{(n+1)} + y^n \left[ \cancel{4}n^2 - \cancel{4}n + \cancel{4}n + 1 \right] = 0$$

$$y^{n+2} \cdot x^2 + (2n+1) x y^{(n+1)} + y^n \left[ \cancel{4}n^2 + 1 \right] = 0$$

$$\boxed{\therefore x^2 y^{n+2} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0}$$