Maths Assignment

17/ENG04/087

EEE 381

Elect / Elect.

1) $y = e^{x+x^2}$

Using the chain rule / function of a fuction.

$y = e^u$, $y = e^u$

$u = x + x^2$ $\quad dy/dx = e^u$ $\therefore du/dx = 1 + 2x$

$dy/du \cdot du/dx = [2x + 1]e^u$

$\cancel{dy/dx = \ldots}$

$\therefore dy/dx = (2x + 1)e^{(x+x^2)}$

$y' = (2x+1)(e^{x+x^2})$

$y''\left(d^2y/dx^2\right) = [2x+1][2x+1(e^{x^2+x})] + 2[e^{x^2+x}]$

$y'' = (2x+1)[(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$y'' = y'(2x+1) + 2(y)$ .... prove b.) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using leibnitz's theorem

$w_1^n = y''$ $\quad u_1 = 1$ $\quad u^0 = y''$

$u_1' = 0$ $\quad u^n = y^{n+2}$

$w_1^n = y^{n+2}(1) + n y^{n+1}(0) + \cdots\cdots$

$w_1^n = y^{n+2}$

Expanding $-y'(2x+1)$

$w_2^n = -2xy'$ $\quad w_3 = -y'$

$w_2^n = -2xy'$

$u^0 = -2x$ $\quad u = y'$

$u' = -2$ $\quad u' = y''$

$u'' = 0$ $\quad u^n = y^{n+1}$

$\therefore w_2^n = u^n u + n u^{n-1} u' + \frac{n(n-1)}{2} u^{n-2} u'' + \cdots\cdots$

$w_2^n = y^{n+1}(-2x) + n y^n(-2) + \frac{n(n-1)}{2} \times y^{n-1}(0)$

$w_2^n = -2x y^{n-1} - 2n y^n$

$w_3^n = -y'$

$u^0 = y'$ $\quad u = -1$

$u' = y''$ $\quad u' = 0$

$u^n = y^{n+1}$

$w_3^n = y_{n+1}(-1) + 0 = -y^{n+1}$

$w_4^n = 2y$

$u = y \qquad v = -2$

$u' = y' \qquad v' = 0$

$u^n = y^n$

$w_4^n = 2y^n$

Adding all the results we have.

$w_1^n + w_2^n + w_3^n + w_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$ .... proven

2.) $y = x^3 e^{4x}$ $\Rightarrow$ determine $y(5)$

$u = e^{4x} \qquad v = x^3$

$u' = 4e^{4x} \qquad v' = 3x^2$

$u'' = 16e^{4x} \qquad v'' = 6x$

$u^n = 4^n e^{4x} \qquad v''' = 6, \quad v^{iv} = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v^2}{2!} + \frac{n(n-1)(n-2)u^{n-3}}{3!}$

$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)4^{n-2}e^{4x}6x}{2!} +$

$\frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4x} 6.$

$$y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + (n-1)n\, 4^{n-2}\, 3 e^{4x} x + n(n-1)(n-2)4^{n-3} e^{4x} \ldots$$

$$y^n = 4^{n-3} e^{4x}\left[4^3 x^3 + n4^2 \times 3x^2 + n(n-1)4 \times 3x + n(n-1)(n-2)\right]$$

$$y^n = 4^{n-3} e^{4x}\left[64x^3 + n\,48x^2 + n(n-1)12x + n(n-1)(n-2)\right]$$

$$y^5 = 4^{5-3} e^{4x}\left[64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)\right]$$

$$y^5 = 16e^{4x}\left[64x^3 + 240x^2 + 240x + 60\right]$$

$$y^2 = \left[1024 e^{4x} x^3 + 3840x^2 e^{4x} + 3840 e^{4x} x + 960 e^{4x}\right]$$

3.) If $x^2 \, d^2y/dx^2 + x\, dy/dx + y = 0$, show that $x^2 y^{n+2} + (2n+1)x\, y^{(n+1)} + (n^2+1)y^n = 0$.

Taking the given equation and differentiating n times, we have.

If $w = x^2 y''$ then $w^{(n)}$ will be $y^{(n+2)} x^2 + n y^{(n+1)} 2x +$

$w = xy'$ then $w^{(n)} = y^{(n+1)} x + n y^{(n)} 1 + 0 \ldots$

$w = y$ then $w^{(n)} = y^{(n)}$

$$\therefore \left[x^2 \, d^2y/dx^2 + x\, dy/dx + y\right]^n =$$

$$x^2 y^{(n+2)} + [2n+1]\, x y^{(n+1)} + (n^2+1) y^{(n)} = 0.$$