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No 1

1. $y = e^{x^2+x}$

let $x^2+x = u$

$y = e^u$; $\frac{dy}{du} = e^u$

$u = x^2+x$; $\frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (2x+1)e^u$

$= (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = 2x+1$, $\frac{du}{dx} = 2$

$v = e^{x^2+x}$, $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = (2x+1)y' + 2y$

from y'',

$y^n = (2x+1)y^{(n-1)} + 2(n-1)y^{(n-2)}$

$\therefore y^{(n+2)} = (2x+1)y^{(n+2-1)} + 2(n+2-1)y^{(n-2+2)}$

$= (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$ //

No 2

(i) If $y = x^3 e^{4x}$, find $y^{(5)}$

let $y = uv$

where, $v = x^3$

$u = e^{4x}$

from leibnitz theorem,

$$y^{(n)} = \sum_{r=0}^{n} {}^nC_r\, u^{(n-r)} v^r$$

$$y^{(5)} = u^{(n)}v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)}$$

$$+ \frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} u^{(n-5)} v^{(5)} + \dots$$

$v^{(1)} = \frac{dv}{dx} = 3x^2$, $v^{(2)} = \frac{d^2v}{dx^2} = 6x$, $v^{(3)} = \frac{d^3v}{dx^3} = 6$, $v^{(4)} = 0$, $v^{(5)} = 0$

recall $u^{(n)} = 4^n e^{4x}$

$u^{(1)} = 4e^{4x}$, $u^{(2)} = 16e^{4x}$, $u^{(3)} = 64e^{4x}$, $u^{(4)} = 256e^{4x}$, $u^{(5)} = 1024 e^{4x}$

$$\therefore y^{(5)} = u^{(5)}v + 5u^{(4)}v^{(1)} + \frac{5(5-1)}{2\times1} u^{(3)}v^{(2)} + \frac{5(5-1)(5-2)}{3\times2\times1} u^{(2)}v^{(3)}$$

$$+ \frac{5(5-1)(5-2)(5-3)}{4\times3\times2\times1} u^{(1)}v^{(4)} + \frac{5(5-1)(5-2)(5-3)(5-4)}{5\times4\times3\times2\times1} u^{(0)}v^{(5)}$$

$$y^{(5)} = 1024e^{4x}(x^3) + 5\times256e^{4x}(3x^2) + \frac{20}{2}(64e^{4x})(6x) +$$

$$\frac{60}{6}(16e^{4x})(6) + \frac{120}{24}(4e^{4x})(0) + \frac{120}{120}(e^{4x})(0)$$

$$= 1024x^3e^{4x} + 5(768x^2e^{4x}) + 10(384xe^{4x}) + 10(96e^{4x}) + 0 + 0$$

$$= 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$$

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No 2 contd

(ii) Treating each term, we differentiate $x^2y'' + xy' + y = 0$ using leibnitz theorem :

for x^2y'', let $u = y''$ and $v = x^2$

$x^2y'' = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{2!}y^{(n)}2 + 0 \ldots$

for xy', let $u = y'$ and $v = x$

$xy' = y^{(n+1)}x + ny^{(n)} + 0 + \ldots$

for y, $y = y^{(n)}$

$\therefore$, $x^2y'' + xy' + y = 0$

$\Rightarrow x^2y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)}$

collecting like terms,

$\Rightarrow x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2-n+n+1)y^{(n)}$

$\Rightarrow x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$\therefore$, $[x^2y'' + xy' + y]^n = 0$ is

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$ //

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