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17/ENG05/041
mechatronics

1)a) $y = e^{x+x^2}$

using the chain rule/ function of a function

$y = e^u$ $\qquad$ $y = e^u$

$u = x + x^2$ $\qquad$ $dy/du = e^u$

$\frac{du}{dx} = 1 + 2x$

$dy/du \times \frac{du}{dx} = (2x+1)e^u$

$dy/dx = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x+x^2})$

$y'' = (2x+1)((2x+1)e^{x^2+x}) + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$\therefore y'' = y'(2x+1) + 2(y)$

b) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

using leibnitz theorem

$w_1^n = y^{n+2}$, $\quad U_1 = 1 \quad u^0 = y''$

$U_1' = 0 \quad u^n = y^{n+2}$

$w_1^n = u^n U + n u^{n-1} U' + \frac{n(n-1)}{2!} u^{n-2} U'' + \cdots$

$w_1^n = y^{n+2}(1) + n y^{n+1}(0) + \cdots$

$w_1^n = y^{n+2}$

expanding $\quad -y'(2x+1)$

$w_2^n = -2y'$ $\qquad$ $w_3 = -y'$

$w_2^n = -2xy'$

$U^{(0)} = -2x \qquad u = y'$

$U' = -2 \qquad u' = y''$

$U'' = 0 \qquad u^n = y^{n+1}$

$w_2^n = u^n U + n u^{n-1} U' + \frac{n(n-1)}{2!} u^{n-2} U'' + \cdots$

$w_2^n = y^{n+1}(-2x) + n y^n(-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$w_2^n = -2xy^{n+1} - 2ny^n$

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$u^0 = y'$ $\quad v = -1$

$u' = y''$ $\quad v' = 0$

$u^n = y^{n+1}$

$w_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$w_4^n = -2y$ $\quad u = y$ $\quad v = -2$

$u' = y'$ $\quad v' = 0$

$u^n = y^n$

$w_4^n = -2y^n$

Adding all results

$$w_1^n + w_2^n + w_3^n + w_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n(y^{n+1}) - 2y^n = 0$$

$$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$$

$$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$$

2) $y = x^3 e^{4x}$

$y^{(5)} = ?$

$u = e^{4x}$ $\quad v = x^3$

$u' = 4e^{4x}$ $\quad v' = 3x^2$

$u'' = 16e^{4x}$ $\quad v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad v''' = 6$

$v^{iv} = 0$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3$$

$$y^n = 4^n e^{4x} x^3 + n4^{n-1}e^{4x}3x^2 + \frac{n(n-1)}{2!}4^{n-2}e^{4x}6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3}e^{4x}6$$

$$y^n = 4^n e^{4x}x^3 + n4^{n-1}e^{4x}3x^2 + n(n-1)4^{n-2}3x + n(n-1)(n-2)4^{n-3}e^{4x}$$

$$y^n = 4^{n-3}e^{4x}[4^3x^3 + n4^2 \cdot 3x^2 + n(n-1)4 \times 3x + n(n-1)(n-2)]$$

$$y^n = 4^{n-3}e^{4x}[64x^3 + n48x^2 + n(n-1)12x + n(n-1)(n-2)]$$

$$y^5 = 4^{5-3}e^{4x}[64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)]$$

$$y^5 = 16e^{4x}[64x^3 + 240x^2 + 240x + 60]$$

$$\therefore y^5 = [1024e^{4x}x^3 + 3840x^2e^{4x} + 3840e^{4x}x + 960e^{4x}]$$

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b) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y^{n+2} + (2n+2)xy^{(n+1)} + (n^2+1)y^n = 0$

$x^2y'' + xy' + y = 0$

$\omega_1 \quad \omega_2 \quad \omega_3$

$\omega_1 = x^2y''$

$v = x^2, \quad v' = 2x, \quad v'' = 2$

$u = y^2 \quad u' = y^3 \quad u^2 = y^4$

$\omega_1^n = \sum_{r=0}^{2} {}^nC_r\, u^{(n-r)} v^r$

$= u^n v^0 + nu^{n+1}v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2$

$= y^{n+2}x^2 + ny^{n+1}\, 2x + \frac{n(n-1)}{2} y^n 2$

$x^2y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n$

$\omega_2 = xy' \quad v = x \quad v' = 1$

$u = y' \quad u' = y^2$

$u^n = y^{(n+1)}$

$\omega_2^n = u^n v + nu^{n-1}v'$

$= y^{(n+1)}x + ny^n \cdot 1$

$= xy^{n+1} + ny^n$

$\omega_3 = y$

$u = y, \quad u^n = y^n$

$= u^n v = y^n \cdot 1 = y^n$

$\omega_1^n + \omega_2^n + \omega_3^n = 0$

$x^3y^{n+2} + 2nxy^{n+2} + n(n-1)y^n + xy^{n+2} + ny^n + y^n = 0$

$x^2y^{n+2} + x(1+2n)y^{n+1} + (n(n-1) + n + 1)y^n = 0$

$x^2y^{n+2} + x(1+2n)y^{n+1} + (n^2-n+1+1)y^n = 0$

$x^2y^{n+2} + x(2n+1)y^{n+1} + (n^2+1)y^n = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

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