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MECHANICAL ENGINEERING.
16/ENG06/028.
ENG 381

ASSIGNMENT.

1.) If $y = e^{x^2+x}$
Show that $y'' = y'(2x+1) + 2y$ and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

2) Using the Leibnitz theorem, given that,
i) $y = x^3 e^{4x}$, determine y^5.
ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; Show that

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$.

SOLUTION.

1.) $y = e^{x^2+x}$
$\ln y = x^2 + x$.
Differentiating both sides.
$\frac{1}{y}\frac{dy}{dx} = 2x+1$

$\frac{dy}{dx} = y(2x+1)$.

$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx}$.

$v = (2x+1)$ $\quad dv/dx = 2$.
$u = y$ $\quad \frac{du}{dx} = \frac{du}{dy} \cdot \frac{dy}{dx}$

$= 1 \cdot \frac{dy}{dx} = \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot \frac{dy}{dx} + 2y$

$\frac{d^2y}{dx^2} = (2x+1)\frac{dy}{dx} + 2y$

$y'' = (2x+1)y' + 2y$

$y'' - (2x+1)y' \mp 2y = 0$

$y^{(2)} - (2x+1)y^{(1)} - 2y = 0$

~~From~~ ~~For~~ $y^{(2)}$

nth derivative $y^{(n+2)}$

~~For~~ $(2x+1)y'$

$u = y' \qquad u^{n} = y^{n+1} \qquad u^{(n-1)} = y''$

$v = 2x+1 \qquad v' = 2 \qquad v^{(2)} = 0$

Using Leibnitz theorem

$y^{(n)} = u^{(n)} + nu^{(n-1)}v^{(1)} + \ldots$

For $2y$

nth derivative $= 2y^{n}$.

$w_1^{(n)} = y^{(n+2)}$

$w_2^{(n)} = y^{(n+1)}(2x+1) + y^{n}2n$.

$w_3^{(n)} = 2y^{n}$.

$y^{(n+2)} - y^{(n+1)}(2x+1) - y^{n}2n - 2y^{n} = 0$.

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^{n} + 2y^{n}$.

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{n}(n+1)$.

2.) $y^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)} + \dfrac{n(n-1)u^{(n-2)}v^{2}}{2!} + \dfrac{n(n-1)(n-2)u^{(n-3)}v^{3}}{3!}$

$u = e^{4x} \qquad\qquad v = x^{3}$

$u^{n} = 4^{n}e^{4x} \qquad\qquad v^{(1)} = 3x^{2}$

$u^{(n-1)} = 4^{(n-1)}e^{4x} \qquad\qquad v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)}e^{4x} \qquad\qquad v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)}e^{4x} \qquad\qquad v = 0$

$u^{n} = 4^{5}e^{4x} = 1024e^{4x}$

$u^{(n-1)} = 4^{(5-1)}e^{4x} = 256e^{4x}$

$u^{(n-2)} = 4^{(5-2)} e^{4x} = 64 e^{4x}$

$u^{(n-3)} = 4^{(5-3)} e^{4x} = 16 e^{4x}$

$y^{(5)} = 1024 e^{4x} x^3 + 5 \times 256 e^{4x} \times 3x^2 + \frac{5(5-1) \times 64 e^{4x}}{2!} + \frac{6x + 5(5-1)(5-2) \times 16 e^{4x}}{3!}$

$y^{(5)} = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + \frac{2680 e^{4x} x}{2!} + \frac{5760 e^{4x}}{3!}$

$y^{(5)} = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4}x + 960 e^{4x}$.

$y^{(5)} = e^{4x} (1024 x^3 + 3840 x^2 + 3840 x + 960)$.

ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

$x^2 y'' + x y' + y = 0$.

$x^2 y^{(2)} + x y^{(1)} + y = 0$.

For $x^2 y^{(2)}$

$u = y^{(2)}$ $\quad u^{(n)} = y^{(n+2)}$ $\quad u^{(n-1)} = y^{(n+1)}$ $\quad u^{(n-2)} = y^{(n)}$

$v = x^2$ $\quad v^{(1)} = 2x$ $\quad v^{(2)} = 2$ $\quad v^{(3)} = 0$.

For $x y'$

$u = y'$ $\quad u^{(n)} = y^{(n+1)}$ $\quad u^{(n-1)} = y^{(n)}$ $\quad u^{(n-2)} = y^{(n-1)}$.

$v = x$ $\quad v^{(1)} = 1$ $\quad v^{(2)} = 0$

For y.

$u^{n} = y^{(n)}$.

Applying Leibnitz theorem

$y^{(n)} = u^{(n)} + n u^{(n-1)} v^{(1)} + \frac{n(n-1) u^{(n-2)} v^2}{2!}$

$w_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1) y^{(n)}}{2!} + \frac{n(n-1)(n-2)}{3!} y^{(n-1)} \cdot 0$

$w_2^{(n)} = y^{(n+1)} \cdot x + n y^{(n)} \cdot 1 + \frac{n(n-1) y^{(n-1)}}{2!} \cdot 0$

$w_3^{(n)} = y^{(n)}$.

$w_1^{(n)} + w_2^{(n)} + w_3^{(n)} = 0$.

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$.

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^{(n)}(n^2-n+n+1) = 0$.

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + (n^2+1)y^n = 0$