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Mechanical Engineering
17/ENG06/108
ENG 381
Engineering Mathematics III

(1) if $y = e^{x^2+x}$
Show that
$y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^u$; $u = x^2 + x$

$\frac{dy}{dx} = e^u$, $\frac{du}{dx} = 2x+1$

$\therefore \frac{dy}{dx} = 2x+1 \cdot e^u$

$\therefore y' = (e^u) \cdot 2x+1$
$y' = (2x+1)(e^{x^2+x})$

$\frac{d^2y}{dx^2} = 2x+1 (2x+1 \cdot [e^{x^2+x}]) + e^{x^2+x} \cdot 2$

$y'' = 2x+1 [y'] + 2(y)$

$\therefore y'' = y'(2x+1) + 2y$ //

For $y'' \rightarrow y^{(n+2)}$
$y'(2x+1)$: $v = 2x+1$ $u^n = y^{n+1}$
$v' = 2$

$2y \rightarrow 2y'$

$$y^{(n+2)} = y^{n+1} \cdot 2x+1 + n\,[y^n 2] + 2y^n$$

$$y^{(n+2)} = (2x+1)\,y^{(n+1)} \cdot 2(n+1)\,y^n$$

Question 2

$y = x^3 e^{4x}$, determine y^5

<u>Solution</u>

where $v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x}$

$v''' = 6$ $\quad u^n = 4^n e^{4x}$

$$\therefore\ y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!}$$

$$y^n = 4^n e^{4x} \cdot x^3 + n\,[4^{n-1} e^{4x} \cdot 3x^2] + \frac{n(n-1)\,[4^{n-2} e^{4x} \cdot 6x]}{2!} + \frac{n(n-1)(n-2)\,[4^{n-3} e^{4x} 6]}{3!}$$

$$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 n\,[4^{n-1} e^{4x}] + n(n-1)3x\,[4^{n-2} e^{4x}] + n(n-1)(n-2)\,[4^{n-3} e^{4x}]$$

Therefore.

$$y^5 = 4^5 e^{4x} \cdot x^3 + 3x^2 \cdot 5\,[4^{5-1} e^{4x}] + 5(5-1)3x\,[4^{5-2} e^{4x}] + 5(5-1)(5-2)\,[4^{5-3} e^{4x}]$$

$$\left[\ y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}\ \right]$$

(3) If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$; show that

$$x^2 y^{n+2} + (2n+1)\,x y^{(n+1)} + (n^2+1)\,y^n = 0$$

Solutions

$x^2y'' \Rightarrow u'' = y^{n+2}$ $\quad v = x^2$

$v' = 2x$

$v'' = 2$

$y'' = y^{n+2} \cdot x^2 + n y^{n+1}\, 2x + n(n-1)\, y^n \cdot 2 / 2!$

$xy' \Rightarrow u^n = y^{n+1}$ $\quad v = x$

$v' = 1$

$y' = y^{n+1} \cdot x + n y^n$

$y \Rightarrow y^n$

$\therefore \; y^{n+2} \cdot x^2 + n y^{n+1} \cdot 2x + n(n-1) y^n + y^{n+1} \cdot x + n y^n \cdot 2 + y^n = 0$

$y^{n+2} \cdot x^2 + x(2n+1) y^{n+1} + (n^2 - n) y^n + n y^n \cdot 2 + y^n = 0$

$y^{n+2} \cdot x^2 + (2n+1)\, x y^{(n+1)} + y^n [n^2 - n + n + 1] = 0$

$y^{n+2} \cdot x^2 + (2n+1)\, x y^{(n+1)} + y^n [n^2 + 1] = 0$

$[\; \therefore \; x^2 y^{n+2} + (2n+1)\, x y^{(n+1)} + (n^2+1)\, y^n = 0 \;]$