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16/ENG04/031
Elect/Elect Engr.
ENG 381 Assignment.

i.) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$.

$u = 2x+1$; $u' = 2$

$v = e^{x^2+x}$; $v' = (2x+1)e^{x^2+x}$.

$y'' = uv' + vu'$

$y'' = (2x+1)[(2x+1)e^{x^2+x}] + e^{x^2+x}\cdot 2$

but $y' = (2x+1)e^{x^2+x}$ and $y = e^{x^2+x}$.

$\therefore y'' = y'(2x+1) + 2y$.

$y'' - y'(2x+1) - 2y = 0$

As $A = y''$

$u = y''$, $u^n = y^{n+2}$

$v = 1$, $v' = 0$

$y^n = u^n v + n u^{n-1} v'$

$y^n = y^{n+2}\cdot 1 + n y^{n+1}\cdot 0$

$y^n = y^{n+2} = A'$.

$y^{n+2}x^2 + ny^{n+1}2x + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$.

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2-n+n+1) = 0$.

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2+1) = 0$.

ii.) $x^2y'' + xy' + y = 0$

$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v^2}{2!} + \frac{n(n-1)(n-2)}{3!}\cdot u^{n-3}v^3 + \dots$

$y = uv$

let $A = x^2y''$, $u = y''$; $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$; $v''' = 0$.

$y^n = y^{n+2}x^2 + ny^{n+1}2x + \frac{n(n-1)y^n}{2!}\times 2 + \frac{n(n-1)(n-2)y^{n-1}}{3!}\times 0$

A':

$y^n = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1)$

let $B = x y'$

$u = y'$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$y^n = u^{n+1} x + (n y^n \times 1) + \frac{n(n-1)}{2} \times y^{n-1} \times 0.$

B':

$y^n = y^{n+1} \times x + n y^n.$

let $C = y$

C':

$y^n = y^n$

$\Rightarrow A + B + C = 0$

$\Rightarrow A' + B' + C' = 0$

$B = y'(2x+1)$

$u = y'$, $u^n = y^{n+1}$

$v = 2x+1$, $v' = 2$, $v'' = 0$

$y^n = y^{n+1}(2x+1) + n y^n . 2$

$y^n = y^{n+1}(2x+1) + 2n y^n$

let $C = 2y$

$u = y$, $u^n = y^n$

$v = 2$, $v' = 0$

$y^n = 2y^n + n y^{n-1} \times 0$

$y^n = 2y^n$

$A - B - C = 0$

$A' - B' - C' = 0$

$y^{n+2} - [y^{n+1}(2x+1) + 2n y^n] - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2n y^n + 2y^n.$

$y^{n+1} = y^{n+1}(2x+1) + y^n(2n+2).$

$\therefore y^{n+1} = (2x+1) y^{n+1} + 2(n+1) y^n$

2) i)

$u = e^{4x} \quad , \quad y^n = 4^n e^{4x}$

$v = x^3 \quad , \quad v' = 3x^2 \, , \; v'' = 6x \quad v''' = 6 \quad v^{iv}$

$$y^n = 4^n e^{4x} . x^3 + n4^{n-1} e^{4x} (3x^2) + \frac{n(n-1)4^{n-2} e^{4x} \times 6x}{2!} + \frac{n(n-1)(n-2)4^{n-3} e^{4x} \times 6}{3!}$$

$+ \ldots$

$y^n = 4^n e^{4x} x^2 + 3x^2 \times n \times 4^{n-1} \times e^{4x} + 3x(n^2 - n)4^{n-2} e^{4x} + n(n^2 - 3n + 2)4^{n-3} \times e^{4x}.$

$y^n = 4^n e^{4x} x^2 + 3x^2 \times n \times 4^{n-1} e^{4x} + 3x(n^2 - n)4^{n-2} \times e^{4x} + (n^3 - 3n^2 + 2n)4^{n-3} \times e^{4x}.$

$y^5 = 1024 e^{4x} x^2 + 3.256 \times 5x^2 \times e^{4x} + 3 \times (5^2 - 5) \times 4^{(n-2)} \times e^{4x} \times x + (5^3 - 3.5^2 + 2.5) \times 4^{n-3} \times e^{4x}$

$y^5 = 1024x^2 e^{4x} + 3840x^2 \times e^{4x} + 3840x e^{4x} + 960 e^{4x}.$

$y^5 = 64 e^{4x} x^3 \left[16 + \frac{60}{x} + \frac{60}{x^2} + \frac{15}{x^3} \right].$