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Course : ENG 381

1)a) $y = e^{x+x^2}$

Using the Chain rule
$y = e^u$, $dy/du = \cancel{eu}\ e^u$
$u = x + x^2$, $\frac{du}{dx} = 1 + 2x$

$dy/du \times du/dx = (2x+1)e^u$
$dy/dx = (2x+1)e^{(x+x^2)}$
$\therefore y' = (2x+1)(e^{(x+x^2)})$

$y'' = \left(\frac{d^2y}{dx^2}\right) = (2x+1)((2x+1)e^{x^2+x}) + 2(e^{x^2+x})$
$y'' = (2x+1)\{(2x+1)\}e^{x^2+x}) + 2(e^{x^2+x})$
where
$e^{x^2+x} = y$
and $y'' = y'(2x+1) + 2(y)$ proven

b) $y'' = y'(2x+1) + 2y$
$y'' - y'(2x+1) - 2y = 0$
Using lebnits theorem
$W_1^n = y''$, $V_1 = 1$ $u^0 = y''$
$V_1 = 0$ $u^n = y^{n+2}$
$W_1^n = U^n v + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \dots$

$\therefore W_1^n = y^{n+2}$

Expanding $= y^n(2x+1)$

$W_2^n = -2xy'$ $\qquad W_3 = \cancel{x} - y'$

$W^n_2 = -2xy'$

$V^{(0)} = -2x \;,\; U = y'$

$V' = -2 \;;\; U' = y''$

$V'' = 0 \;,\; U'' = y^{n+1}$

$$W^n_2 = U^n + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \dots$$

$$W_2^n = y^{n+1}(-2x) + ny^n(-2) + \frac{n(n-1)}{2!}y^{(n-1)}(0)$$

$$W_2^n = -2xy^{n+1} - 2ny^n$$

$W_3^n = -y'$

$U^0 = y' \;;\; V = -1$

$U' = y'' \qquad V' = 0$

$U^n = y^{n+1}$

$$W_3^n = y^{(n+1)}(-1) + 0 = -y^{(n+1)}$$

$W_4^n = -2y$ / $\quad U = y \;,\; V = -2$

$U' = y' \;,\; V' = 0$

$U^n = y^n$

$$W_4^n = -2y^n$$

Adding

$$W^n_1 + W^n_2 + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2xy^n - y^{(x+1)} - 2y^n = 0$$

$$y^{n+2} + 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$$

$$y^{n+2} = 2xy^{n+1}\; 2ny^n + y^{n+1} + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n \quad // \text{ Proven}$$

2) Determine $y^{(5)}$ if $y = x^3 e^{4x}$

Solu

$y = x^3 e^{4x}$

$u = e^{4x}$ $\qquad$ $v = x^3$

$u' = 4e^{4x}$ $\qquad$ $v' = 3x^2$

$u'' = 16e^{4x}$ $\qquad$ $v'' = 6x$

$u^n = 4^n e^{4x}$ $\qquad$ $v^{iv} = 0$

$\cancel{y = u^n v^2 + n y^{n+1}}$

$y = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} \cdot \cancel{e^{4x}} \cancel{6x\, nCn}$

$\cdot e^{4x} 6x + \frac{n(n-1)(n-2) \cdot 4^{n-3} e^{4x} \cdot 6}{6}$

$y^5 = 16^{4x}(64x^3 + 240x^2 + 240x + 60)$