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Mechatronics Engineering
ENG 381
Engineering Mathematics

1 $y = e^{x^2+x}$

$y' = (2x+1)\, e^{u^2+u}$

$u = 2x+1$, $v = e^{x^2+x}$

$\frac{dy}{dx} = 2$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}$

$u'' = u'(2x+1) + 2x$

$u'' = u(2x+1) + 2y$

$u'' = u'(2x+1) - 2y = 0$

$= y''$

$w = u''$

$u^n = u^{n+2}$ $v = 1$

$u^n = u^{n+2}$ $v' = 0$

$w^n = {}^nC_0 u^{n-0} u^0 + {}^nC_1 u^{n-1} v'$

$= y^{n+2} + 0$

$= y^{n+1}$

$w = y'(2x+1)$

$u = y'$ $v = 2x+1$

$u^n = y^{n+1}$ $u = 2$

$w_2^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 v^{n-1} v'$

$= u^n v + n u^{n-1} v'$

$= y^{n+1}(2x+1) + 2ny^n$

$w_1 = 2y$

$u = y$ $v = 2$ $v' = 0$

$u' = y$

$u^n = y^n$

$$w_3^n = {}^nC_0 u^{n-0} v^0$$
$$= u^n v$$
$$w_3^n = 2y^0$$
$$y'' - y'(2x+1) - 2y = 0$$
$$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y = 0$$
$$y^{n+2} - y^{n+1}(2x+1) + 2ny^n) - 2y = 0$$
$$y^{n+2} - y^{n+1}(2x+1) - 2y_n(n+1) = 0$$
$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

2 $y = x^3 e^{4x}$

$$y^5 = u^5 v + 5u^{(5-1)} v' + \frac{5(5-1)u^{(5-2)} v^{(2)}}{x} + \frac{5(5-1)(5-1)}{3!}$$
$$+ \frac{5(3-1)(5-2)(5-3)u'v}{4!} + uv^{(3)}$$

$$y^5 = u^5 v + 5u^4 v' + \frac{20u^3 v^3}{2!} + 60u^2 v^3 + \frac{\quad v^4}{4!} + uv^5$$
$$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + uv^5$$

when, $u = e^{4x}$ $\quad v = x^3$

$u^5 = 4^5 e^{4x}$ $\quad v = 3x^2$

$u^5 = 1024e^{4x}$ $\quad v^{(2)} = 6x$

$u^4 = 4^4 e^{4x} = 256e^{4x}$ / $v^3 = 6$

$u^3 = 4^3 e^{4x} = 64e^{4x}$ $\quad v^4 = 0$

$u^2 = 4^2 e^{4x} = 16e^{4x}$ $\quad v^5 = 0$

$u' = 4' e^{4x} = 4e^{4x}$

$$y^{(5)} = 1024e^{4x} x^3 + 5 \cdot 256 e^{4x} \cdot 3x^2 + 10 \cdot 64 e^{4x} + 6x +$$
$$10 \cdot 16e^{4x} \cdot 6 + 5 \cdot 16e^{4x} \cdot 6 + e^{4x} 0$$
$$y^5 = 64e^{4x}\{16x^3 + 60x^2 + 60x + 15\}$$

II $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

The equation can be rewritten as

$x^2 y'' + xy' + y = 0$

Taking each term and differentiating n times using

Leibnitz theorem

$F_1 W_1 = x^2 y^n$

let $u = y^{(n)}$ $\quad V = x^2$

$u^2 = y^{(n+2)}$ $\quad V^1 = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad V^2 = 2$

$u^{(n-1)} = y^n$ $\quad V^3 = 0$

$\therefore W^{(n)}$

from $y^{(n)} = u^n V + n V^{(n-1)} V^{(1)} + \frac{n(n-1)x^{(n-2)} V^2}{2!} \dots$

$w_1^{(n)} = y^{(n+2)} x^2 + ny^{(n+1)} 2x + \frac{n(n-1)y^{n-2}}{2} + 0$

$W_1^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$

if $w_2 = xy'$

let $u = y^{(1)}$ $\quad V = x$

$u^{(n)} = y^{(n+1)}$ $\quad V^{(1)} = 1$

$u^{(n-1)} = u^{(n)}$ $\quad V^{(2)} = 0$

$w_1^n = y^{(n+1)} - x + ny^{(n)} + 0$

if $w_3 = y$

$u = y$

$u^{(n)} = y^{(n)}$

$w_3^{(n)} = y^{(n)}$

Therefore $w_1^{(n)} + w_2^n + w_3^n = 0$

$\Rightarrow x^2 y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n + y^{(n+1)}x + ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$