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COURSE: ENG 381 [Assignment 2]

1. $y = e^{x^2+x}$

let $u = x^2+x$, $y = e^u$

$\frac{dy}{du} = e^u$ $\frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = (2x+1) \cdot e^u$

$\cancel{\frac{dy}{dx}} = \& \; y' = (2x+1) \cdot e^u$ $y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[(2x+1) \cdot e^{x^2+x}\right]$

Using product rule

$u = 2x+1$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = (2x+1) e^{x^2+x}$

$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot (2x+1) \cdot e^{x^2+x} + e^{x^2+x} \cdot 2$

recall, $y' = (2x+1) \cdot e^{x^2+x}$, $y = e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot (2x+1) \cdot e^{x^2+x} + 2e^{x^2+x}$

$y'' = (2x+1) \cdot y' + 2y$

Hence, $y'' = y'(2x+1) + 2y$

For $\omega_1^{(n)}$ $u = y^{(2)}$ $v = 1$

$u^{(n)} = y^{(n+2)}$ $v^{(1)} = 0$

From Leibniz

For ω_2^{n}

$u = y^{(1)}$ $v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $v^{(1)} = 2$

$$u^{(n-1)} = y^{(n)} \quad v^{(2)} = 0$$

For $\omega_2^{\,n}$

$$u = y \quad v = 2$$

$$u^{(n)} = y^{(n)} \quad v^{(1)} = 0$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + n2y^{(n)} + 2y^{(n)}$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$$

Hence, $y^{(n+2)} = y^{(n+1)}(2x+1) + 2(n+1)y^{(n)}$

2. Using Leibniz theorem

$y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^{(n)} = 4^{(n)} e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^{(4)} = 0$

$$y^{(n)} = u^{(n)} v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)u^{(n-2)}}{2!} v^{(2)} + \frac{n(n-1)(n-2)u^{(n-3)}}{3!} v^{(3)} + \cancel{\frac{n(n-1)(n-2)(n-3)u^{(n-4)}}{4!} v^{(4)}}$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + \frac{5 \cdot 4}{} 4^3 e^{4x} \cdot 6x + \frac{5 \cdot 4 \cdot 3}{3!} 4^2 e^{4x} 6$$

$$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$$

$$y^{(5)} = \cancel{4^5 e^{4x}}\ 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii, $\frac{x^2 d^2y}{dx} + \frac{x\,dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

For $w_1^{(n)}$

$u = y^2$ $\quad v = x^2$

$u^{(n)} = y^{(n+2)}$ $\quad v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v^{(3)} = 0$

For $w_2^{(n)}$

$u = y^{(1)}$ $\quad v = x$

$u^{(n)} = y^{(n+1)}$ $\quad v^{(1)} = 1$

$u^{(n-1)} = y^{(n)}$ $\quad v^{(2)} = 0$

For $w_3^{(n)}$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v = 0$

$$w_1^{(n)} = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{n-2} + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} u^3$$

$$w_1^{(n)} = y^{(n+2)} x^2 + n(y^{n+1})\, 2x + \frac{n(n-1) y^n}{2!} \times 2$$