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Problem Statement

(1) IF $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$
and hence, Prove that
$y^{(n+2)} = (2x+1)y^{(n-1)} + 2(n+1)y^{n}$

(2) Using the Leibnitz theorem, given that
i. $y = x^3 e^{4x}$, determine $y^{(5)}$
ii. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$.

Solution

(1) $y = e^{x^2+x}$, $y^{(0)} = e^{x^2+x}$

$y^{(1)} = (2x+1)e^{x^2+x}$

$y^{(2)} = (2x+1)(2x+1)e^{x^2+x} + 2e^{x^2+x}$

$y^{(2)} = y^{(1)}(2x+1) + 2y^{(0)}$

$\therefore\ y'' = y'(2x+1) + 2y$

(ii) $y^{(2)} = y^{(1)}(2x+1) + 2y^{(0)}$

$0 = y^{(1)}(2x+1) + 2y^{(0)} - y^{(2)}$

$\underbrace{y^{(2)}}_{W_1} - \underbrace{y^{(1)}(2x+1)}_{W_2} - \underbrace{2y^{(0)}}_{W_3} = 0$

$W_1 = y^{(2)}$

$V^{(0)} = 1$ $\quad U^{(0)} = y^{(2)}$

$U^{(n)} = y^{(n+2)}$

$W_2 = y^{(1)}(2x+1)$

$V^{(0)} = (2x+1)$ $\quad U^{(0)} = y^{(1)}$

$V^{(1)} = 2$ $\quad U^{(1)} = y^{(2)}$

$U^{(n)} = y^{(n+1)}$

$W_3 = 2y^{(0)}$

$V^{(0)} = 2 \quad U^{(0)} = y^{(0)}$

$U^{(n)} = y^{(n)}$

$W_1^{(n)} - W_2^{(n)} - W_3^{(n)} = 0$

$y^{(n+2)} - [y^{(n+1)}(2x+1) + 2ny^{(n)}] - 2y^{(n)} = 0$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2ny^{(n)} + 2y^{(n)}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{(n)}$

$U \cdot V$

2) b $y = x^3 e^{4x}$

$V^{(0)} = x^3, \quad V^{(1)} = 3x^2, \quad V^{(2)} = 6x, \quad V^{(3)} = 6$

$U^{(0)} = e^{4x}, \quad U^{(1)} = 4e^{4x}, \quad U^{(2)} = 16e^{4x}, \quad U^{(3)} = 64e^{4x}, \quad U^{(4)} = 256e^{4x}, \quad U^{(5)} = 1024e^{4x}$

$y^{(5)} = {}^5C_0\, 1024e^{4x}x^3 + {}^5C_1\, 256e^{4x} \cdot 3x^2 + {}^5C_2\, 64e^{4x} \cdot 6x + {}^5C_3\, 16e^{4x} \cdot 6$

$y^{(5)} = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 160e^{4x}$

$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 160)$

$y^{(5)} = 160e^{4x}(32/5\, x^3 + 24x^2 + 24x + 1)$

A b $x^2y'' + xy' + y = 0$

$x^2y^{(2)} + xy^{(1)} + y^{(0)} = 0$

$W_1 + W_2 + W_3$

$W_1 = x^2y^{(2)}$

$V^{(0)} = x^2, \quad V^{(1)} = 2x, \quad V^{(2)} = 2$

$U^{(0)} = y^{(2)} \quad U^{(1)} = y^{(3)}, \quad U^{(2)} = y^{(4)}, \quad U^{(n)} = y^{(n+2)}$

$W_2 = xy^{(1)}$

$V^{(0)} = x \qquad U^{(0)} = y^{(1)}$

$V^{(1)} = 1 \qquad U^{(1)} = y^{(2)}$

$U^{(n)} = y^{(n+1)}$

$W_3 = y^{(0)}$

$V^{(0)} = 1 \quad U^{(0)} = y^{(0)}$

$U^{(n)} = y^{(n)}$

$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$

$y^{(n+2)} \cdot x^2 + ny^{(n+1)} \cdot 2x + \dfrac{n(n-1)y^{n} \cdot \cancel{2}}{\cancel{2}} + y^{(n+1)} \cdot x + ny^{(n)} + y^{(n)} = 0$

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$$y^{(n+2)} \cdot x^2 + 2xn\,y^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + y^{(n)}(n+1) = 0$$

$$x^2y^{(n+2)} + 2xn\,y^{(n+1)} + xy^{(n+1)} + n(n-1)y^{(n)} + y^{(n)}(n+1) = 0$$

$$x^2y^{(n+2)} + x(1+2n)y^{(n+1)} + y^{(n)}(n^2-n+n+1) = 0$$

$$x^2y^{(n+2)} + x(1+2n)y^{(n+1)} + (n^2+1)y^{(n)} = 0$$