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16/ENG03/031
Civil ENGINEERING
ENG 381

$y = e^{x^2+x}$

let $u = x^2+x$, $y = e^u$

$\frac{dy}{du} = e^u$ $\frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{dy}{du}$

$\frac{dy}{dx} = e^u \cdot (2\cancel{x}+1)$

$y' = (2x+1) \cdot e^u$

$y' = (2x+1) e^{x^2+x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[(2x+1) \cdot e^{x^2+x}\right]$

Using product rule

$u = 2x+1$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\frac{dv}{dx} = (2x+1) \cdot e^{x^2+x}$

$\frac{d^2y}{dx^2} = \frac{v\,du}{dx} + u\frac{dv}{dx}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot (2x+1) \cdot e^{x^2+x} + e^{x^2+x}(2)$

but $y' = (2x+1) \cdot e^{x^2+x}$

$y = e^{x^2+x}$

$y'' = (2x+1) \cdot y' + 2y$

$y^{(n)}$

for w.

$u = y^{(2)}$ $v = 1$

$u^{(n)}$ $y^{(n+2)}$ $v^{(1)} = 0$

for $W_2^{(n)}$

$u = y^{(1)}$ $\quad$ $v = 2x + 1$

$u^{(n)} = y^{(n+1)}$ $\quad$ $v^{(1)} = 2$

$u^{(n-1)} = y^{(n)}$ $\quad$ $v^{(1)} = 0$

for $W_3^{(n)}$

$u^{(n)} = y^{(n)}$ $\quad$ $v = 2$

$u^{(n)} = y^{(n)}$ $\quad$ $v^{(1)} = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + n\,\cancel{2y}^{(n)} + 2y^{(n)}$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}\,(n+1)$

⑤ Using leibnitz theorem.

$y = x^3 e^{4x}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u^{(n)} = 4^{(n)} e^{4x}$ $\quad$ $v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad$ $v^2 = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad$ $v^3 = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad$ $v^4 = 0$

$y(n) = u^{(n)}v + n u^{(n-1)} v^{(1)} + \frac{n}{2!}(n-1) u^{(n-2)} v^{(2)} + \dots$

$\cancel{y^{(5)} = 4^{5}}$

$y^{(5)} = 4^{(5)} e^{4x} \cdot x^3 + 5(4^{(4)}) e^{4x} \cdot 3x^2 + \frac{5(5-1)}{2!} 4^3 e^{4x} \cdot 6x +$

$\frac{5(5-1)(5-2)\,4^2 e^{4x} \cdot 6}{3!}$

$y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$

$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

ii) $x^2 \frac{d^2y}{dx^2} + \frac{x\,dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

for $W_1^{(n)}$

$u = y^{(2)}$ $\quad$ $v = x^2$

$u^{(n)} = y^{(n+2)}$ $\quad$ $v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad$ $v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad$ $v^{(3)} = 0$

for $W_2^{(n)}$

$u = y^{(1)}$ $\quad$ $v = x$

$u^{(n)} = y^{(n+1)}$ $\qquad v^{(1)} = 1$

$u^{(n-1)} = y^{(n)}$ $\qquad v^{(2)} = 0$

for $w_3^{(n)}$

$u = y$ $\qquad v = 1$

$u^{(n)} = y^{(n)}$ $\qquad v = 0$

$$w_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{\cancel{2}!} y^{(n)} \cancel{2}$$

$$w_2^{(n)} = y^{(n+1)} x + n y^{(n)}$$

$$w_3^{(n)} = y^{(n)}$$

$$w_1^{(n)} + w_2^{(n)} + w_3^{(n)} = 0$$

$$0 = y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^{(n)} + y^{(n+1)} x$$

$$+ n y^{(n)} + y^{(n)}$$

$$0 = y^{(n+2)} x^2 + x y^{(n+1)} (2(n)+1) + 1\, y^{(n)} (n^2 - n + n + 1)$$

$$0 = y^{(n+2)} x^2 + x y^{(n+1)} (2n+1)\ y^{(n)} (n^2 + 1)$$

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