SOKUNBI FARID CIVIL ENGINEERING 16/ENG03/051 ENG381

1(a) $y = e^{x^2+x}$ $y'' = y'(2x+1)+2y$

assuming

$u = x^2+x$ , $y = e^u$

$\frac{du}{dx} = 2x+1$ $\frac{dy}{du} = ue^u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$\frac{dy}{dx} = (2x+1) \times ae^u$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$y' = \frac{dy}{dx} = (2x+1)e^{x^2+x}$

$y'' = \frac{d^2y}{dx^2} = [(2x+1)(2x+1)e^{x^2+x}] + 2(e^{x^2+x})$

recall

$y = e^{x^2+x}$ $y' = (2x+1)(e^{x^2+x})$

$\therefore y'' = y'(2x+1)+2y$

1b $y'' = y'(2x+1)+2y$

$y'' - y'(2x+1) - 2y = 0$

From leibnitz theorem

$W_1^n = y''$

$V_1 = 1$, $V_1' = 0$

$u^0 = y''$, $u^n = y^{n+2}$

$W_1^n = u^nV + nu^{n-1}V' + \frac{n(n-1)}{2!}u^{n-2}V'' + \dots$

$W_1^n = y^{n+2}(1) + ny^{(n+1)}(0) + \dots$

$\therefore W_1^n = y^{(n+2)}$

$-y'(2x+1) = -2xy' - y'$

$W_2 = -2xy'$ $W_3 = -y'$

$W_2^n = -2xy'$

$V^0 = -2x,\ V' = -2,\ V'' = 0$

$u^0 = y',\ u' = y'',\ u^n = y^{n+1}$

$W_2^n = u^n V + n u^{n-1} V' + \frac{n(n-1)}{2!} u^{n-2} V'' + \ldots$

$W_2^n = -2xy^{(n+1)} - 2ny^n$

$W_3^n = -y'$

$u^0 = y',\ u' = y'',\ u^n = y^{(n+1)}$

$V^0 = -1,\ V' = 0$

$W_3^n = y^{(n+1)}(-1) + 0 = -y^{(n+1)}$

$W_4 = -2y$

$u^0 = y,\ u' = y',\ u^n = y^n$

$V^0 = -2,\ V' = 0$

$W_4^n = -2y^n$

$u = y,\ u' = y',\ u^n = y^n$

$V^0 = -2,\ V' = 0$

$\therefore W_1^n + W_2^n + W_3^n + W_4^n$

$= y^{(n+2)} - 2xy^{(n+1)} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$\therefore y^{(n+2)} = (2x+1)\, y^{(n+1)} + 2(n+1)y^n$

(2) (a) $y = x^3 e^{4x}$, determine $y^{(5)}$

$n = 5$

$V = x^3,\ u = e^{4x}$

$V^0 = x^3,\ V' = 3x^2,\ V^2 = 6x,\ V^3 = 6;\ V^4 = 0$

$u^0 = e^{4x},\ u' = 4e^{4x},\ u^2 = 16e^{4x},\ u^3 = 64e^{4x}$

$u^n = 4^n e^{4x}$

$\therefore u^5 = 4^5 e^{4x}$

$u = 1024 e^{4x}$

$y^n = u^n x^3 + n u^{n-1} 3x^2 + \frac{n(n-1)}{2} u^{n-2} \cdot 6x + \frac{n(n-1)(n-2)}{6} \times u^{n-3} \times 6$

$y^5 = 1024e^{4x} x^3 + 5 \cdot 256\, x^2 + 5(5-1)64e^{4x} + 5(5-1)(5-2)16e^{4x}$

$y^{(5)} = 1024e^{4x}x^3 + 3840x^2 + 1280e^{4x} + 960e^{4x}$

(2b) $x^2y'' + xy' + y = 0$

let,

$w_1 = x^2y''$, $w_2 = xy'$, $w_3 = y$

$w_1 = x^2y''$

$v = x^2$, $v' = 2x$, $v'' = 2$

$u = y^2$, $u' = 2y$, $u'' = y$

$\therefore u_n^{(n)} = y^{(n+2)}$

$$w_1^{(n)} = \sum_{r=0}^{n} {}^nC_r \, u^{(n-r)} v^r$$

$$w_1^n = u^{(n)}v^{(0)} + nu^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)}v^{(2)}$$

$$w_1^{(1)} = y^{(n+2)}x^2 + ny^{(n+1)}2x + \frac{n(n-1)}{\cancel{2}} \times y^{(n)} \times \cancel{2}$$

$$w_1^{(n)} = x^2y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^{(n)}$$

$w_2 = xy'$

$v = x$, $v' = 1$

$u = y^{(1)}$, $u' = y''$

$u^{(n)} = y^{(n+1)}$

$$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$

$x^2y'' + xy' + y = 0$

$w_2^{(n)} = u^{(n)}v + nu^{(n-1)}v^{(1)}$

$w_2^{(n)} = y^{(n+1)}x + ny^{(n)} \cdot 1$

$w_2^{(n)} = xy^{(n+1)} + ny^{(n)}$

$w_3 = y$

$u = y$, $v = 1$

$u^{(n)} = y^{(n)}$

$w_3 = u^n v = y^{(n)} \cdot 1 = y^{(n)}$

$$\therefore W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2nxy^{(n+1)} + n(n-1)y^{(n)} + xy^{(n+1)} + ny^{(n)} + y^{(n)}$$

$$x^2 y^{(n+2)} + x(1+2n)y^{(n+1)} + (n(n-1)+n+1)y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(2n+1)y^{(n+1)} + (n^2+1)y^{(n)} = 0$$