1) $y = e^{x^2+x}$

$u = x^2 + x \qquad y = e^u$

$\cancel{\frac{du}{dx}} \Rightarrow \frac{du}{dx} = 2x+1$

$y = e^u \quad \frac{dy}{du} = e^u \times \frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \times 2x+1$

$= 2x+1 e^{x^2+x}$

using Product rule (Lebinitz theory)

$\frac{d^2y}{dx^2} = u'v + v'u$

$= 2e^{x^2+x} + (2x+1)(2x+1)e^{(x^2+x)}$

let $e^{x^2+x} = y, \therefore = 2y + (2x+1)\frac{dy}{dx}$

let $(2x+1)e^{x^2+x} = \frac{dy}{dx}$,

$\therefore \frac{d^2y}{dx^2} = y'' = 2y + \frac{dy}{dx}(2x+1)$

2) $y = x^3 e^{4x}$, find y^5

$y^5 = 5C_0 u^5 v^0 + 5C_1 u^{(5-1)} v' + {}^5C_2 u^{(5-2)} v'' + {}^5C_3 u^{(5-3)} v'''$
$+ {}^5C_4 u^{(5-4)} v^{iv} + {}^5C_5 u^{(5-5)} v^v$

$u = e^{4x} \qquad v = x^3$

$u' = 4e^{4x} \qquad v' = 3x^2$

$u'' = 16e^{4x} \qquad v'' = 6x$

$u''' = 64e^{4x} \qquad v''' = 6$

$u^{iv} = 256e^{4x} \qquad v^{iv} = 0$

$u^v = 1025e^{4x} \qquad v^v = 0$

$\therefore y^5 = [1025e^{4x} \cdot x^3] + 5[256e^{4x} \cdot 3x^2] + 10[64e^{4x} \cdot 6x] +$
$10[16e^{4x} \cdot 6] + 5[4e^{4x} \cdot 0] + [e^{4x} \cdot 0]$

$y^5 = 1025x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$

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iii) $x^2y'' + xy' + y = 0$

Show $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' + xy' + y = 0$

let $w_1 = x^2y''$

$w_2 = xy'$

$w_3 = y$

$w_1 = x^2y''$

$v = x^2 \qquad u = y''$

$v' = 2x \qquad u' = y'''$

$v'' = 2 \qquad u'' = y^{(4)}$

$u^n = y^{(n+2)}$

$w_1 = {}^nC_0u^nv^0 + {}^nC_1u^{(n-1)}v' + {}^nC_2u^{(n-2)}v''$

$= {}^nC_0y^{(n+2)}x^2 + {}^nC_1y^{(n+1)}2x + {}^nC_2y^{(n)}2$

$= x^2y^{(n+2)} + 2nxy^{(n+1)} + \frac{n(n-1)}{2}y^{(n)}2$

$w_1 = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$

$w_2 = xy'$

$v = x \qquad u = y'$

$v' = 1 \qquad u' = y''$

$v'' = 0 \qquad u'' = y'''$

$u^n = y^{(n+1)}$

$w_2 = {}^nC_0u^nv^0 + {}^nC_1u^{(n-1)}v' + {}^nC_2 \ldots$

$= {}^nC_0y^{(n+1)}x + {}^nC_1y^{(n)}\cdot 1$

$w_2 = xy^{(n+1)} + ny^{(n)}$

$w_3 = 1\cdot y$

$v = 1 \qquad u = y$

$v' = 0 \qquad u' = y', \; u^n = y^n$

$w_3 = {}^nC_0u^nv^0 + \ldots$

$= {}^nC_0y^{(n)}\cdot 1 = y^{(n)}$

$\therefore \; W_1 + W_2 + W_3 = 0$

$$0 = [x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^{(n)}] + [x y^{(n+1)} + n y^{(n)}] + y^{(n)}$$

$$0 = x^2 y^{(n+2)} + (2xn + x) y^{(n+1)} + (n^2 - n + n + 1) y^{(n)}$$

$$0 = x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 + 1) y^{(n)}$$