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Elect/Elect.
300Level.
16/ENG04/086.

1. Problem statement.

IF $y = e^{x^2} + x$.

Show that

$y'' = y'(2x+1) + 2y$ and hence

$y^{(n+2)} = (2x+1)y^{(n-1)} + 2(n+1)y^{n}$.

2. Using Leibnitz theorem, given that

(i) $y = x^3 e^{+2x}$, determine $y^{(5)}$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. Show that $x^2 y^{(n+2)} + (2n+1) y^{(n+1)} + (n^2+1) y^{(n)} = 0$.

Sol

(i) $y = e^{x^2} + x$, $y^{(0)} = e^{x^2} + x$.

$y^{(1)} = (2x+1) e^{x^2+x}$,

$y^{(2)} = (2x+1)(2x+1) e^{x^2+x} + 2e^{x^2+x}$

$y^{(2)} = y^{(1)}(2x+1) + 2y^{(0)}$.

$y'' = y'(2x+1) + 2y$.

(ii) $y^{(2)} = y^{(1)}(2x+1) + 2y^{(0)}$.

$0 = y^{(1)}(2x+1) + 2y^{(0)} - y^{(1)}$,

$y^{(2)} - y^{(1)}(2x+1) - 2y^{(0)} = 0$

W1 ↑ W2 ↑ ↑ W3

$W_1 = y^{(2)}$

$V^{(0)} = V^{(0)} = y^{(2)}$

$V^{n} = y^{n+2}$.

$W_2 = y^{(1)}(2x+1)$

$y^{(0)} = y^{(1)}$

$y^{(1)} = y^{(2)}$

$y^{(n)} = y^{(n)}$.

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$w_3 = 2y^{(n)}$

$v^{(0)} = 2 \quad u^{(0)} = y^{(0)} \quad u^{(n)} = y(n)$

$w_1^{(n)} - w_2^{(n)} - w_3^{(n)} = 0$

$y(n+2) - [y(n+1)(2x+1) + 2n\,y(n)] - 2y(n) = 0$

$y(n+2) = (2x+1)\,y(n+1) + 2n\,y(n) + y(n)$

$y(n+2) = (2x+1)\,y(n+1) + 2(n+1)\,y(n)$.

(2) $y = x^3 e^{4x}$

$v^{(0)} = x^3, \quad v^{(1)} = 3x^2, \quad v^{(2)} = 6x, \quad v^{(3)} = 6$

$u^{(0)} = e^{4x}, \quad u^{(1)} = 4e^{4x}, \quad u^{(2)} = 16e^{4x}, \quad u^{(3)} = 64e^{4x}$

$y^{(5)} = {}^5C_0\, 1024 e^{4x}\, x^3 + {}^5C_1\, 256 e^{4x}\, 3x^2 + {}^5C_2\, 64 e^{4x} \cdot 6x + {}^5C_3\, 16 e^{4x} \cdot 6$.

$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 160)$

$y^{(5)} = 160 e^{4x}(32/5\, x^3 + 24x^2 + 24x + 1)$

(i) $x^2 y'' + x y' + y = 0$.

$x^2 y(2) + x y(1) + y(0) = 0$

$w_1 = x^2 y(n)$.

$v^{(0)} = x^2, \quad v(1) = 2x, \quad v(2) = 2$

$u^{(0)} = y(2) \quad u^{(1)} = y(3), \quad u^{(2)} = y^{(4)}$

$w_2 = x y(1)$.

$v^{(0)} = x \qquad u^{(0)} = y'$

$v^{(1)} = 1 \qquad u^{(1)} = y(2)$

$w_3 = y(0)$

$v(0) = 1, \quad u^{(n)} = y^{(0)}$.

$w_1(n) + w_2(n) + w_3(n) = 0$

$\Sigma y.\ y(n+2) \cdot x^2 + n y(n+1) \cdot 2x +$

$\frac{n(n-1)}{2} y_n \cdot 2 + y(n+1) \cdot x$

$+ n y(n) + y(n) = 0$

$\Rightarrow y(n+2) \cdot x^2 + 2xn\,y(n+1)$

$+ n(n-1)\,y(n) + x y(n+1) + y(n)$

$(n+1) = 0$

$x^2 y(n+2) + 2xn\,y(n+1)$

$x y(n+1) + n(n-1)\,y(n) +$

$(n+1) = 0$.

$\therefore x^2 y(n+2) + x(1+2n)\,y(n+1)$

$+ (n^2+1)\,y(n) = 0$.

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