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$$(1) y = e^{x^2+x}$$

let  $x^2+x = u$

$$\frac{du}{dx} = 2x+1$$

$$y = e^u \quad \frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= e^u (2x+1)$$

$$= (2x+1) e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y'' = u \frac{du}{dx} + v \frac{dv}{dx}$$

$$= (2x+1)(2x+1) e^u + e^{x^2+x} (2)$$

because  $y = e^{x^2+x}$

$$y' = (2x+1) e^u$$

$$\therefore y'' = (2x+1) y' + 2y$$

from  $y''$

$$y^{(n)} = (2x+1) y^{(n-1)} + 2(n-1) y^{(n-2)}$$

$$\therefore y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}$$

$$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}$$



$$= y^{(n+1)} x + n y^{(n)}$$

$$w_3 = y'$$

$$V = 1$$

$$U = y'$$

$$U^{(n)} = y^{(n)}$$

$$w_3 = U^{(n)} V$$

$$w_3 = y^n$$

$$w_1^{(n)} + w_2^{(n)} + w_3^{(n)} = 0$$

$$x^2 y^{(n+2)} + n y^{(n+1)} x + n(n-1) y^n + y^{(n+1)} x + n y^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n^2 - n + n + 1) y^{(n)} = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$$

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$$y = x^3 e^{4x}$$

$$\sum_{r=0}^n {}^n C_r u^{(n-r)} v^r$$

$$v = x^3, v^{(1)} = 3x^2, v^{(2)} = 6x, v^{(3)} = 6, v^{(4)} = 0, v^{(5)} = 0$$

$$u = e^{4x}, u^{(1)} = 4e^{4x}, u^{(2)} = 16e^{4x}, u^{(3)} = 64e^{4x}, u^{(4)} = 256e^{4x}$$

$$y^{(5)} = u^{(5)} v + 5u^{(5-1)} v^{(1)} + 5C_2 u^{(5-2)} v^{(2)} + 5C_3 u^{(5-3)} v^{(3)} + 5C_4 u^{(5-4)} v^{(4)}$$

$$= u^{(5)} v + 5u^{(4)} v^{(1)} + 5C_2 u^{(3)} v^{(2)} + 5C_3 u^{(2)} v^{(3)} + 5C_4 u^{(1)} v^{(4)}$$

$$= 1024e^{4x} x^3 + 3840e^{4x} x^2 + 3840e^{4x} x + 960e^{4x} + 0$$

$$y^{(5)} = e^{4x} (1024x^3 + 3840x^2 + 3840x + 960)$$

$$(11) x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + x y' + y = 0$$

$$x^2 y'' = w_1$$

$$v = x^2, v^{(1)} = 2x, v^{(2)} = 2$$

$$u = y'', u^{(1)} = y^{(3)}, u^{(2)} = y^{(4)}, u^{(n)} = y^{(n+2)}$$

$$w_1^{(n)} = \sum_{r=0}^n {}^n C_r u^{(n-r)} v^r$$

$$= u^{(n)} v + n u^{(n-1)} v^{(1)} + {}^n C_2 u^{(n-2)} v^{(2)}$$

$$= u^{(n)} v + n u^{(n-1)} v^{(1)} + \frac{n(n-1) y^{(n+2)}}{2}$$

$$w_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n$$

$$x y' = w_2$$

$$v = x, v^{(1)} = 1$$

$$u = y^{(1)}, u^{(1)} = y^{(2)}, u^{(n)} = y^{(n+1)}$$

$$w_2 = u^{(n)} v + n u^{(n-1)} v^{(1)}$$