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16/ENG05/006
Mechatronics.

Engineering Maths.

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

take $\cos t = 0$

$\Rightarrow \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$x = Ae^{kt}$

$\frac{dx}{dt} = kAe^{kt} = kx$

$\frac{d^2x}{dt^2} = k^2Ae^{kt} = k^2x$

Substituting $\frac{dx}{dt}, \frac{d^2x}{dt^2}$ & x ~~into~~

$k^2x + 5kx + 6x = 0$

divide through by x

$k^2 + 5k + 6 = 0$ (Auxiliary solution)

$k^2 + 3k + 2k + 6 = 0$

$\cancel{k^2+3k}$ -

$k(k+3) + 2(k+3) = 0$

$(k+3)(k+2)$

$k = -3$ or -2

$x = Ae^{-3t} + Be^{-2t} \longrightarrow$ General solution.

Taking the standard form of the RHS

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

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$$\frac{d^2x}{dt^2} = -A\cos t \cancel{+} B\sin t$$

Sub $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$ & x

$[-A\cos t - B\sin t] + 5[-A\sin t + B\cos t] +$

$6[A\cos t + B\sin t] = \cos t$

$\sin t(-B - 5A + 6B) + \cos t[-A + 5B + 6A] = \cos t$

Equate the Coefficients

$-5A + 5B = 0$

$5A + 5B = 1$

$A = \cancel{+}\frac{1}{\cancel{10}}\ \ 2\frac{1}{10}$

$-5A + 5B = 0$

$5B = 5A$

$B = A = \frac{1}{10}$

P.I $\Rightarrow x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$x = \frac{1}{10}(\cos t + \sin t)$ (Particular integral).

Recall: C.S $\longrightarrow x = Ae^{-3t} + Be^{-2t}$

When $t = 0$; $x = 0.1$ & $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t}$$

Subst x & t

$0.1 = Ae^{-3(0)} + Be^{-2(0)}$

$0.1 = A + B$ —— (1)

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Substituting $\frac{dx}{dt}$ & $t = 0$

$$0 = -3Ae^{-3(0)} - 2Be^{-2(0)}$$

$$0 = -3A - 2B \quad \text{--- (ii)}$$

$$A = 0.1 - B \quad \text{--- (iii)}$$

$$0 = -0.3 + 3B - 2B$$

$$B = 0.3$$

From eq. (iii) $A = 0.1 - B$

$$A = 0.1 - 0.3$$

$$A = -0.2$$

$\therefore$ General solution $\rightarrow x = -0.2e^{-3t} + 0.3e^{-2t}$

Complete general solution $\rightarrow x = -0.2e^{-3t} + 0.3e^{-2t} + \frac{1}{10}\left[\cos t + \sin t\right]$