AMIESIMAKA MENTONI
16/ENG07/006
ENG 381

1. $$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

assuming $\cos t = 0$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m = -2 \quad \text{or} \quad m_2 = -3$$

Complementary function $x = Ae^{-2t} + Be^{-3t}$

Particular Integration

$f(x) = \cos t \qquad x = C\cos t + D\sin t)$

$$\frac{dx}{dt} = -(\cos t + (-D\sin t)$$

$$-(\cos t - D\sin t + 5(-C\sin t + D\cos t)$$

$$+6(C\cos t + D\sin t) = \cos t$$

$$-(\cos t - D\sin t - 5C\sin t + 5D\cos t + 6\cos t +$$

$$6D\sin t$$

$$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$$

$$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t$$

By comparing coefficients

$5D + 5C = 1 \quad \ldots (1)$

$5D - 5C = 0 \quad \ldots (2)$

$$10D = 1$$

$$D = \frac{1}{10}$$

Subs 1/10 for D in eqn ①

$$5D + 5C = 1$$

$$\frac{5}{10} + 5C = 1$$

$$5C = 1 - 0.5$$

$$5C = 0.5$$

$$C = \frac{0.5}{5} = \frac{1}{10}$$

∴ Particular Integration

$$+6Q = 1/10 \cos t + 1/10 \sin t$$

General solution = CF + PI

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

When $t = 0$, $x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0.1 - 0.1 = 0$$

$$A = -B \quad \cdots ③$$

When $t = 0$, $x = 0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$$

$$0.1 = A + B + \frac{1}{10}$$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$$

$$0 = -2A - 3B + 0.1$$

$$2A + 3B = 0.1 \quad \cdots \quad (4)$$

$$A = -B$$

Subs $-B$ for A in eq 4.

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$$A = -B$$

$$A = -0.1$$

Particular solution

$$x = 0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

OR

$$x = \frac{-e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{1}{10}\cos t + \frac{\sin t}{10}$$

MATLAB

Command window

```
clear
clc
close all
syms t
x(t) = 0.1 + [(exp(-3*t) - (exp(-2*t)) + cos(t) + sin(t)]
tn = [0.001 15]
xd = subs(x(t), tn]
figure()
plot (tn, xd)
axis tight
grid on
grid minor
```