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Petroleum Engineering
ENR 282 Assignment 1

(a). A differential equation is a relationship between an independent Variable, x , and a dependent Variable y , and one or more derivatives of y with respect to x .

(b)

i) Second Order equation

ii) It is second order equation because it has two arbitrary constants.

iii) $y = Ae^{-4x} + Be^{-6x}$ — (1)

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$
 — (2)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$
 — (3)

$$A = \frac{y - Be^{-6x}}{e^{-4x}}$$
 — (4)

Substitute eqn 4 into eqn 2.

$$\frac{dy}{dx} = -4 \left(\frac{y - Be^{-6x}}{e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$= -4y + 4Be^{-6x} - 6Be^{-6x}$$
$$= -4y - 2Be^{-6x}$$
 — (5)

Make B the subject of formula

$$\frac{dy}{dx} = -4y - 2Be^{-6x}$$

$$\frac{dy}{dx} + 4y = -2Be^{-6x}$$

$$\frac{-1}{2e^{-6x}} \left[\frac{dy}{dx} + 4y \right] = B$$
 — eqn 6

Substitute eqn 6 into eqn 4

$$A = \frac{y - Be^{-6x}}{e^{-4x}}$$

$$A = \frac{y - \left[\frac{-1}{2e^{-6x}} \left[\frac{dy}{dx} + 4y \right] \right] e^{6x}}{e^{-4x}}$$

$$A = \frac{y + \frac{dy}{2dx} + 2y}{e^{-4x}} \quad \text{--- eqn 7}$$

Substitute eqn 7 and eqn 6 into eqn 3

$$\begin{aligned} \frac{d^2 y}{dx^2} &= 16Ae^{-4x} + 36Be^{-6x} \\ &= 16 \left[\frac{y + \frac{dy}{2dx} + 2y}{e^{-4x}} \right] e^{4x} + 36 \left[\frac{-1}{2e^{-6x}} \left[\frac{dy}{dx} + 4y \right] \right] e^{-6x} \end{aligned}$$

$$\frac{d^2 y}{dx^2} = 16y + \frac{16dy}{2dx} + 32y - \frac{36dy}{2dx} - 72y$$

$$\begin{aligned} \frac{d^2 y}{dx^2} &= 16y + 8\frac{dy}{dx} + 32y - 18\frac{dy}{dx} - 72y \\ &= -24y - 10\frac{dy}{dx} \end{aligned}$$

$$\frac{d^2 y}{dx^2} + 24y + 10\frac{dy}{dx} = 0$$

--- Differential equation from
 $y = Ae^{-4x} + Be^{-6x}$