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Petroleum Engineering
17/EH/GO7/1008.
CS

1) Define differential Equation and give two examples
Differential Equation is defined as the relation between independent variable and dependent variable and one or more derivative of the dependent variable with respect to the independent variable.

→ $\sin x \frac{dy}{dx} - 3 = 5y$

→ $\frac{d^2y}{dx^2} - \cos x = y^2 + 1$

2) An expression has been obtained for an engineering system to be as given in Equation (i)

$$y = Ae^{-4x} + Be^{-bx} \quad \text{——— (I)}$$

i) What is the order of the differential equation that can be formed from the expression
→ Second-order differential equation

ii) Give a reason for your answer in b(i)
→ This is because the expression is a function with two arbitary constant

iii) Form the differential equation from the expression
→

$$y = Ae^{-4x} + Be^{-bx} \quad \text{——— (1)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-bx} \quad \text{——— (2)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36e^{-bx} \quad \text{——— (3)}$$

from equation ②

$$-4Ae^{-4x} = \frac{dy}{dx} + 6Be^{-6x}$$

$$4Ae^{-4x} = -\left[\frac{dy}{dx} + 6Be^{-6x}\right]$$

$$4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \left[-\frac{dy}{dx} - 6Be^{-6x}\right] \cdot \frac{1}{4e^{-4x}} \quad \text{——— ④}$$

from equation ③ and put A

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = \cancel{16}^{4}\left[\left(-\frac{dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{\cancel{4}\cancel{e^{-4x}}}\right]\cancel{e^{-4x}} + 36Be^{-6x}$$

$$= 4\left(-\frac{dy}{dx} - 6Be^{-6x}\right) + 36Be^{-6x}$$

$$= -4\frac{dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$= -4\frac{dy}{dx} + 12Be^{-6x}$$

$$\therefore B = \left[\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right] \cdot \frac{1}{12e^{-6x}} \quad \text{——— ⑤}$$

Put B in equation ④

$$A = \left(-\frac{dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{4e^{-4x}}$$

$$= \left(-\frac{dy}{dx} - \cancel{6}\left[\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right) \cdot \frac{1}{\cancel{12}_{2}\cancel{e^{-6x}}}\right] \cdot \cancel{e^{-6x}}\right) \cdot \frac{1}{4e^{-4x}}$$

$$= \left[-\frac{dy}{dx} - \left[\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right]\frac{1}{2}\right]\frac{1}{4e^{-4x}}$$

$$A = -\frac{dy}{dx} - \left[\frac{1}{2}\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\right] \cdot \frac{1}{4e^{-4x}}$$

$$= \left[-\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right] \cdot \frac{1}{4e^{-4x}}$$

$$= A = \left[-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right] \cdot \frac{1}{4e^{-4x}} \quad \text{———} \quad (6)$$

Put equation (6) and (5) in (1)

$$y = Ae^{-4x} + Be^{-6x}$$

$$= \left[\left(-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right) \cdot \frac{1}{4\cancel{e^{-4x}}}\right] \cancel{e^{-4x}} + \left[\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right) \cdot \frac{1}{12\cancel{e^{-6x}}}\right] \cancel{e^{-6x}}$$

$$= \left(-\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2}\right) + \left(\frac{1}{12}\frac{d^2y}{dx^2} + \frac{1}{3}\frac{dy}{dx}\right)$$

$$= \left[-\frac{3}{4}\frac{dy}{dx} + \frac{1}{3}\frac{dy}{dx}\right] - \left\{\frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{dy}{dx}\right\}$$

$$= -\frac{5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

$$y = \frac{1}{12}\left[-5\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right]$$