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Petroleum Engineering
17/Eng07/008
CV

① Given that;

$$F = x^2 i + (3x+2)j + \sin x k$$

Find

a) $\frac{dF}{dx}$ b) $\frac{d^2F}{dx^2}$ c) $\left|\frac{dF}{dx}\right|$ d) $\frac{d}{dx}(F.F)$ at $x=1$

Answer

a) $\frac{dF}{dx} = 2xi + 3j + \cos x \dot{k}$

At $x = 1$

$\frac{dF}{dx} = 2(1)i + 3j + \cos(1)\dot{k}$

$= 2i + 3j + 0.9998\dot{k}$

b) $\frac{d^2F}{dx^2} = \frac{d}{dx}\left[\frac{dF}{dx}\right] = 2i - \sin x \dot{k}$

at $x = 1$

$\frac{d^2F}{dx^2} = 2i - \sin(1)\dot{k}$

$= 2i - 0.0175\dot{k}$

c) $\left|\frac{dF}{dx}\right| = 2i + 3j + 0.9998\dot{k}$

$= \left[2^2 + 3^2 + (0.9998)^2\right]^{1/2}$

$= [13.9996]^{1/2}$

$= 3.74$

d) $\frac{d}{dx}(F.F)$

$f \cdot f = (x^2 i + (3x+2)j + \sin x\, k) \cdot (x^2 i + (3x+2)j + \sin x\, i)$

$= x^4 + (9x^2 + 12x + 4) + (\sin^2 x)$

N/B: $i \cdot i = 1$

$j \cdot j = 1$

$k \cdot k = 1$

$\frac{d}{dx}(f \cdot f) = 4x^3 + (18x + 12) + 2\sin x \cos x$

at $x = 1$

$\frac{d}{dx}(f \cdot f) = 4 + 18 + 12 + 2\sin(1)\cos(1)$

$= 4 + 30 + 0.035$

$= 34.035$

(2) If

$r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

determine

a) $\frac{dr}{dt}$ b) $\frac{d^2 r}{dt^2}$ and c) the value of $\left|\frac{d^2 r}{dt^2}\right|$ at $t = 0$

Answer

a) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, j + 6e^{2t}$

at $t = 0$

$\frac{dr}{dt} = (2(0) + 3)i - 6\cos 3(0)j + 6e^{2(0)}$

$= 3i - 6j + 6k$

b) $\frac{d}{dt}\left[\frac{dr}{dt}\right] = 2i + 18\sin 3t\, j + 12e^{2t}$

at $t = 0$

$\frac{d^2 r}{dt^2} = 2i + 18\sin 3(0)j + 12e^{2(0)}$

$= 2i + 12k$

c) $\left|\frac{d^2r}{dt^2}\right| = 2i + 12k$

$= \left[2^2 + 12^2\right]^{1/2}$

$= \left[148\right]^{1/2}$

$= \sqrt{148}$

$= \sqrt{4 \times 37}$

$= \sqrt{4} \times \sqrt{37}$

$= 2\sqrt{37}$

$= \underline{2\sqrt{37}}$