

QUESTION 1 [20 MARKS]

If the maximum percentage absolute error is desired to be 1E-9, using the Newton-Raphson iteration method and initial guess value of 0.5, find the root of the function given in Equation (1.1)

- (a) manually, and
 - (b) with the aid of MATLAB.

$$F(x) = e^{-0.5x} x (4-x) - 2 \quad \dots \dots \dots \text{Equation (1.1)}$$

NB: For the manual solution, use all the values given by the calculator.

Solution

A. Manual solution

$$F(x) = e^{-0.5x} X(4-x) - 2$$

$$F'(x) = e^{-0.5x} (-1) - (4-x) \times 0.5 \times e^{-0.5x} = -e^{-0.5x} \times (1 + 0.5 \times (4-x))$$

$$F'(x) = -e^{-0.5x} X (3-0.5x)$$

Applying the Newton-Raphson iteration method,

$$x_{i+1} = x_i - (F(x_i)/F'(x_i))$$

using an initial guess of $x = 0.5$,

$$x_{i+1} = 0.5 - ((e^{-0.5(0.5)} X (4-(0.5)) - 2) / (-e^{-0.5(0.5)} X (3-0.5(0.5)))) = 0.838890606$$

$$x_{i+1} = 0.838890606 - ((e^{-0.5(0.838890606)} X(4-(0.838890606))-2)/(-e^{-0.5(0.838890606)} X(3-0.5(0.838890606)))) = 0.8849560003$$

$$x_{i+1} = 0.8849560003 - ((e^{-0.5(0.8849560003)} X(4-(0.8849560003))-2)/(-e^{-0.5(0.8849560003)} X(3-0.5(0.8849560003)))) = 0.885708605$$

$$x_{i+1} = 0.885708605 - ((e^{-0.5(0.885708605)} X(4-(0.885708605))-2)/(-e^{-0.5(0.885708605)} X(3-0.5(0.885708605)))) = 0.885708802$$

$$x_{i+1} = 0.885708802 - ((e^{-0.5(0.885708802)} X(4-(0.885708802))-2)/(-e^{-0.5(0.885708802)} X(3-0.5(0.885708802)))) = 0.885708802$$

The absolute error is given as:

$$Ea = |(x_{i+1} - x_i) / x_{i+1}| \times 100$$

$$Ea = |(0.838890606 - 0.5) / 0.838890606| \times 100 = 40.39747299$$

$$Ea = |(0.8849560003 - 0.838890606) / 0.8849560003| \times 100 = 5.205388097$$

$$Ea = |(0.885708605 - 0.8849560003) / 0.885708605| \times 100 = 0.08497204337$$

$$Ea = |(0.885708802 - 0.885708605) / 0.885708802| \times 100 = 2.224207319 \times 10^{-5}$$

$$Ea = |(0.885708802 - 0.885708802) / 0.885708802| \times 100 = 0$$

In tabular form the results are:

i	X_{i+1}	Ea
0	0.5	-
1	0.838890606	40.39747299
2	0.8849560003	5.205388097
3	0.885708605	0.08497204337
4	0.885708802	$2.224207319 \times 10^{-5}$
5	0.885708802	0

B. MATLAB SOLUTION

The screenshot shows the MATLAB R2018a interface. The current folder browser shows several M-files: sclass1.m, sclass2.m, sclass3.m, sclass4.m, sclass41.m, sclass82.m, sclass91.m, Sclass51.m, Sclass52.m, Sclass61.m, Sclass62.m, Sclass71.m, Sclass72.m, Sclass81.m, semilorelectmachines.m, and semiloremath.m. The workspace browser lists variables: Ea, F, H, i, iter, P, x, and xf. The editor window displays the code for ENG382assign2.m:

```
1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - format long g
6 - syms x
7 - F = exp(-0.5*x) * (4-x)^-2;
8 - P = diff(F);
9 - pretty(F)
10 - pretty(P)
11 -
12 - x = 0.5;
13 - for i = 1:10;
14 -     iter(i+1) = i;
15 -     xf(i) = x;
16 -     x = double(subs(x-(F/P)));
17 -     xf(i+1) = x;
18 -     Ea(i+1) = abs((xf(i+1)-xf(i))/xf(i+1))*100;
19 -     if Ea(i+1)<= 1E-9;
20 -         break
21 -     end
22 -
23 - end
24 - H = [iter' xf' Ea']
25 -
```

The command window at the bottom shows the output of the code execution.

Figure 1: Code for the program

The screenshot shows the MATLAB R2018a interface. The top menu bar includes HOME, PLOTS, APPS, EDITOR (selected), PUBLISH, and VIEW. The FILE, NAVIGATE, EDIT, and BREAKPOINTS tabs are also visible. The Current Folder browser shows files like sclass1.m through sclass41.m, sclass82.m, sclass91.m, Sclass51.m, Sclass52.m, Sclass61.m, Sclass62.m, Sclass71.m, Sclass72.m, Sclass81.m, semiloreelectmachines.m, and semiloremath.m. The Editor window displays the code for ENG382assign2.m. The Command Window shows the execution of symbolic calculations for F and P. The Workspace browser lists variables: Ea, F, H, i, iter, P, x, and xf.

```
F =  
- exp(-x/2)*(x - 4) - 2  
  
P =  
(exp(-x/2)*(x - 4))/2 - exp(-x/2)  
/ x \  
- exp| -- | (x - 4) - 2  
 \ 2 /  
  
/ x \  
exp| -- | (x - 4)  
 \ 2 / ----- / x \  
----- - exp| -- |  
 2 \ 2 /  
  
H =  
0 0.5 0  
1 0.838890606045279 40.3974729962571  
2 0.884956000280852 5.20538808945905  
3 0.885708604962403 0.0849720412938465  
4 0.885708802004764 2.2246855836934e-05  
5 0.885708802004777 1.52925215034205e-12
```

Figure 2: RESULTS OF CODE