QUESTION 1 [20 MARKS]

If the maximum percentage absolute error is desired to be 1E-9, using the Newton-Raphson iteration method and initial guess value of 0.5, find the root of the function given in Equation (1.1)

(a) manually, and

(b) with the aid of MATLAB.

F(x) = $e^{-0.5x}$ x (4-x) -2 ……Equation (1.1)

NB: For the manual solution, use all the values given by the calculator.

Solution

A. Manual solution

F(x) = $e^{-0.5x}$ X (4-x) -2

F'(x) = $e^{-0.5x}$ (-1) − (4-x) X 0.5 X $e^{-0.5x}$ = -$e^{-0.5x}$ X (1+0.5 X (4-x))

F'(x) = -$e^{-0.5x}$ X (3-0.5x)

Applying the Newton-Raphson iteration method,

$x_{i+1} = x_i - (F(x_i)/F'(x_i))$

using an initial guess of x =0.5,

x_{i+1} = 0.5 – (($e^{-0.5(0.5)}$ X (4-(0.5)) -2)/ (-$e^{-0.5(0.5)}$ X (3-0.5(0.5)))) = 0.838890606

x_{i+1}=0.838890606 –(($e^{-0.5(0.838890606)}$ X(4-(0.838890606))-2)/(-$e^{-0.5(0.838890606)}$ X(3-0.5(0.838890606)))) = 0.8849560003

x_{i+1}=0.8849560003–(($e^{-0.5(0.8849560003)}$ X(4-(0.8849560003))-2)/(-$e^{-0.5(0.8849560003)}$ X(3-0.5(0.8849560003)))) = 0.885708605

x_{i+1}=0.885708605–(($e^{-0.5(0.885708605)}$ X(4-(0.885708605))-2)/(-$e^{-0.5(0.885708605)}$ X(3-0.5(0.885708605)))) = 0.885708802

x_{i+1}=0.885708802–(($e^{-0.5(0.885708802)}$ X(4-(0.885708802))-2)/(-$e^{-0.5(0.885708802)}$ X(3-0.5(0.885708802)))) = 0.885708802

The absolute error is given as:

Ea = $|(x_{i+1} - x_i) / x_{i+1}|$ X 100

Ea = $|(0.838890606 - 0.5) / 0.838890606|$ X 100 = 40.39747299

Ea = $|(0.8849560003 - 0.838890606) / 0.8849560003|$ X 100 = 5.205388097

Ea = $|(0.885708605 - 0.8849560003) / 0.885708605|$ X 100 = 0.08497204337

Ea = $|(0.885708802 - 0.885708605) / 0.885708802|$ X 100 = $2.224207319 * 10^{-5}$

Ea = $|(0.885708802 - 0.885708802) / 0.885708802|$ X 100 = 0

In tabular form the results are:

i	X_{i+1}	Ea
0	0.5	-
1	0.838890606	40.39747299
2	0.8849560003	5.205388097
3	0.885708605	0.08497204337
4	0.885708802	$2.224207319 * 10^{-5}$
5	0.885708802	0

B. MATLAB SOLUTION

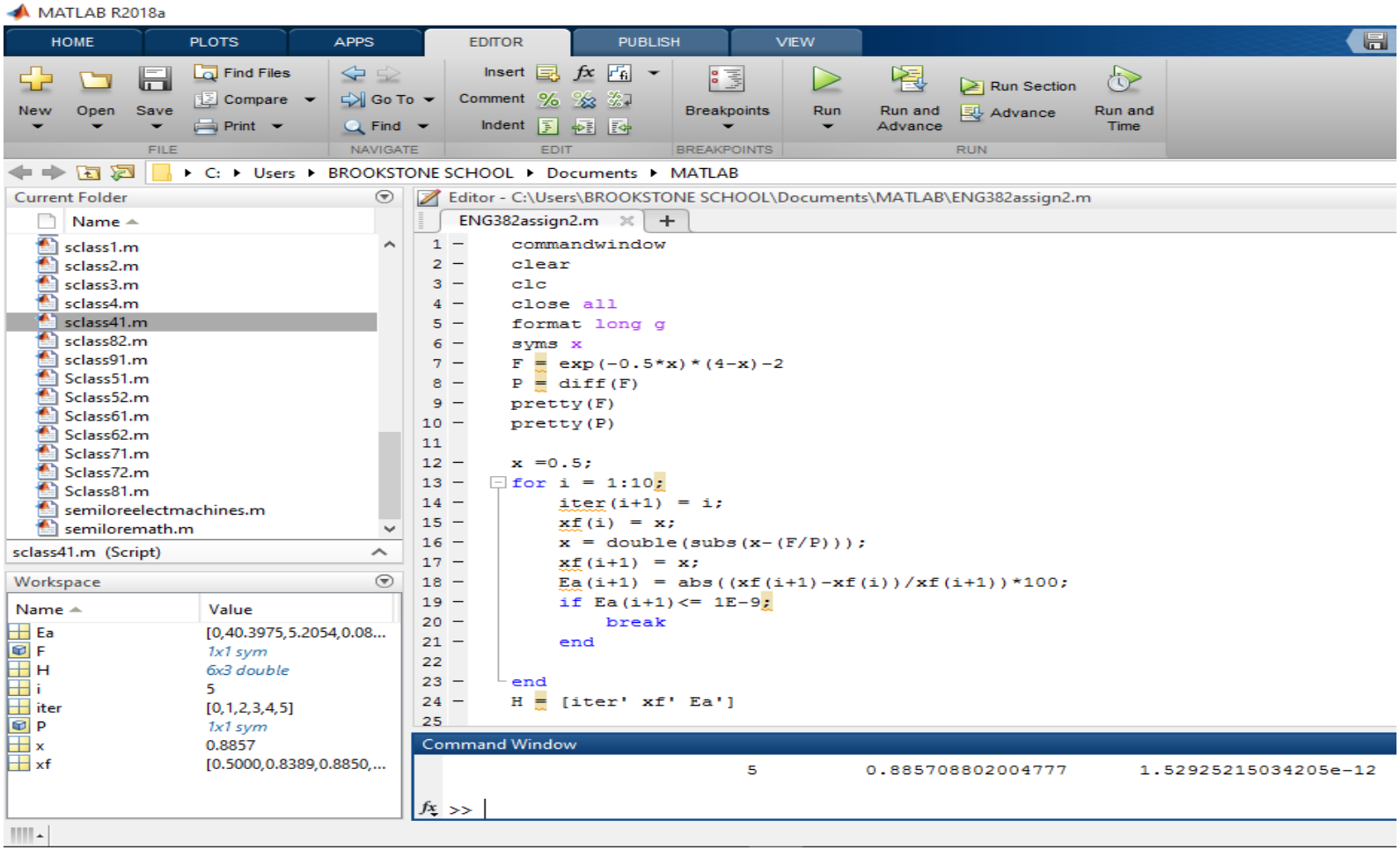

Figure 1: Code for the program

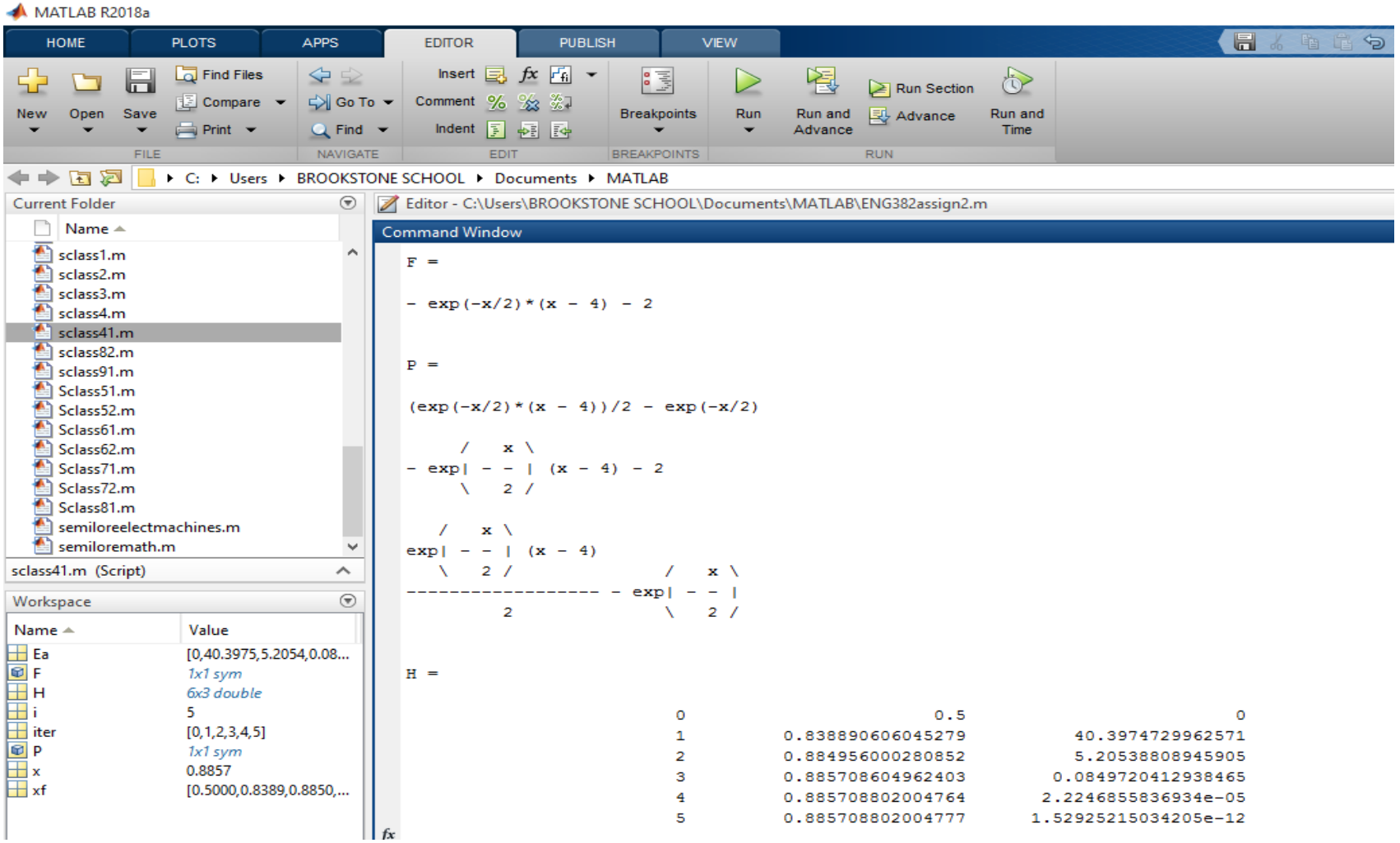

Figure 2: RESULTS OF CODE