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Department :- Computer Engineering

College :- Engineering

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Question 2

If the maximum percentage absolute error is desired to be 1E-9 using the Newton - Raphson iteration method and initial guess value of 0.5, find the root of the function given.

$$f(x) = e^{-0.5x}(4 - x) - 2$$

Solution

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{-0.5x}(4 - x) - 2$$

$$f'(x) = u = e^{-0.5x}, \quad v = 4 - x$$

$$f'(x) = \frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$f'(x) = (4 - x)(-0.5e^{-0.5x}) + e^{-0.5x}(-1)$$

$$f'(x) = 0.5e^{-0.5x}(x - 4) - e^{-0.5x}$$

$$x_{i+1} = x_i - \frac{e^{0.5x}(4 - x) - 2}{0.5e^{-0.5x}(x - 4) - e^{-0.5x}}$$

i	x	ε_a
0	0.5	0
1	0.838890606	40.39747299
2	0.884955809	5.205386019
3	0.885908607	0.084997447061
4	0.885908802	

$$X_{0+1} = \frac{0.5 - e^{-0.5(0.5)}(4-0.5) - 2}{e^{-0.5(0.5)}(0.5-4) - e^{-0.5\times 0.5}}$$

$$= 0.838890606$$

$$\varepsilon_a \% \text{ error} = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

$$= \left| \frac{0.838890606 - 0.5}{0.838890606} \right| \times 100\%$$

$$= 40.39742299$$

$$X_2 = 0.838890606 - \frac{e^{-0.5(0.838890606)}(4 - 0.838890606) - 2}{0.5e^{-0.5(0.838890606)}(0.838890606 - 4) - e^{-0.5(0.838890606)}}$$

$$= 0.884955980 9$$

$$\varepsilon_a = \left| \frac{0.884955980 9 - 0.838890606}{0.884955980 9} \right| \times 100$$

$$= 5.205386019$$

$$X_3 = 0.884955980 9 - \frac{e^{-0.5(0.884955980 9)}(4 - 0.884955980 9) - 2}{0.5e^{-0.5(0.884955980 9)}(0.884955980 - 4) - e^{-0.5(0.884955980 9)}}$$

$$= 0.885708607 1$$

$$\varepsilon_a = \left| \frac{0.885708607 1 - 0.884955980 9}{0.885708607 1} \right| \times 100$$

$$= 0.084974470 61$$

$$x_4 = 0.88520860 71 - \frac{e^{-0.5(0.8857086071}(4 - 0.8857086071) - 2}{0.5e^{-0.5(0.8857086071)}(0.8857086071 - 4) - e^{0.5(0.8857086071)}}$$

$$= 0.88570880 2$$

$$\varepsilon_a = \left| \frac{0.88570880 2 - 0.8857086071}{0.88570880 2} \right| \times 100$$

$$= 2.200497495 \times 10^{-5}$$

$$x_5 = 0.88570880 2 - \frac{e^{-0.5(0.88570880 2)}(4 - 0.88570880 2) - 2}{0.5e^{-0.5(0.88570880 2)}(0.88570880 2 - 4) - e^{0.5(0.88570880 2)}}$$

$$= 0.88570880 2$$

$$\varepsilon_a = \left| \frac{0.88570880 2 - 0.88570880 2}{0.88570880 2} \right| \times 100$$

$$= 0_a$$