

①

OKAFOR MARTINS CHINONSO.

16/ENG 07/018.

PETROLEUM ENGR.

ENG 382

ASSIGNMENT 3.

$$\begin{cases} \bar{T}_1 + \bar{T}_2 - 2\bar{T}_3 + \bar{T}_4 + 3\bar{T}_5 - \bar{T}_6 = 4 \\ 2\bar{T}_1 - \bar{T}_2 + \bar{T}_3 + 2\bar{T}_4 + \bar{T}_5 - 3\bar{T}_6 = 20 \\ \bar{T}_1 + 3\bar{T}_2 - 3\bar{T}_3 - \bar{T}_4 + 2\bar{T}_5 + \bar{T}_6 = -15 \\ 5\bar{T}_1 + 2\bar{T}_2 - \bar{T}_3 - \bar{T}_4 + 2\bar{T}_5 + \bar{T}_6 = -3 \\ 3\bar{T}_1 - \bar{T}_2 + 2\bar{T}_3 + 3\bar{T}_4 + \bar{T}_5 + 3\bar{T}_6 = 16 \\ 4\bar{T}_1 + 3\bar{T}_2 + \bar{T}_3 - 6\bar{T}_4 - 3\bar{T}_5 - 2\bar{T}_6 = -27 \end{cases}$$

The Corresponding Matrix is

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Augmenting the matrix, we have

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

②

To make $a_{21} = 0$
 Row 2 - $\left(\frac{a_{21}}{a_{11}}\right) \times \text{Row 1}$; but $\frac{a_{21}}{a_{11}} = \frac{2}{1} = 2$

For a_{21} : $2 - (2)1 = 0$
 a_{22} : $-1 - (2)1 = -3$
 a_{23} : $1 - (2)(-2) = 5$
 a_{24} : $2 - (2) \times 1 = 0$
 a_{25} : $1 - (2) \times 3 = -5$
 a_{26} : $-3 - (2) \times (-1) = -1$
 a_{27} : $20 - (2) \times (4) = 12$

To make $a_{31} = 0$
 Row 3 - $\left(\frac{a_{31}}{a_{11}}\right) \times \text{Row 1}$; but $\frac{a_{31}}{a_{11}} = \frac{1}{1} = 1$

For a_{31} : $1 - (1) \times 1 = 0$
 a_{32} : $3 - (1) \times 1 = 2$
 a_{33} : $-3 - (1) \times (-2) = -1$
 a_{34} : $-1 - (1) \times 1 = -2$
 a_{35} : $2 - (1) \times 3 = -1$
 a_{36} : $1 - (1) \times (-1) = 2$
 a_{37} : $-15 - (1) \times 4 = -19$

To make $a_{41} = 0$
 Row 4 - $\left(\frac{a_{41}}{a_{11}}\right) \times \text{Row 1}$; but $\frac{a_{41}}{a_{11}} = \frac{5}{1} = 5$

For a_{41} : $5 - (5) \times 1 = 0$
 a_{42} : $2 - (5) \times 1 = -3$
 a_{43} : $-1 - (5) \times (-2) = 9$
 a_{44} : $-1 - (5) \times 1 = -6$
 a_{45} : $2 - (5) \times 3 = -13$
 a_{46} : $1 - (5) \times (-1) = 6$
 a_{47} : $-3 - (5) \times 4 = -23$

(3)

To make $a_{51} = 0$
 Row 5 - $(\frac{a_{51}}{a_{11}}) \times \text{Row 1}$; But $\frac{a_{51}}{a_{11}} = \frac{-3}{1} = -3$
 For a_{51} : $-3 - (-3) \times 1 = 0$
 a_{52} : $-1 - (-3) \times 1 = 2$
 a_{53} : $2 - (-3) \times -2 = -4$
 a_{54} : $3 - (-3) \times 1 = 6$
 a_{55} : $1 - (-3) \times 3 = 10$
 a_{56} : $3 - (-3) \times -1 = 0$
 a_{57} : $16 - (-3) \times 4 = 28$

To make $a_{61} = 0$
 Row 6 - $(\frac{a_{61}}{a_{11}}) \times \text{Row 1}$; But $\frac{a_{61}}{a_{11}} = \frac{4}{1} = 4$
 For a_{61} : $4 - (4) \times 1 = 0$
 a_{62} : $3 - (4) \times 1 = -1$
 a_{63} : $1 - (4) \times -2 = 9$
 a_{64} : $-6 - (4) \times 1 = -10$
 a_{65} : $-3 - (4) \times 3 = -15$
 a_{66} : $-2 - (4) \times -2 = 2$
 a_{67} : $-27 - (4) \times 4 = -43$

Stage 1 (Forward Elimination)

$$\left[\begin{array}{ccccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{array} \right]$$

To make $a_{32} = 0$
 Row 3 - $(\frac{a_{32}}{a_{22}}) \times \text{Row 2}$; But $\frac{a_{32}}{a_{22}} = \frac{-2}{3}$
 For a_{32} : $2 - (\frac{-2}{3}) \times -3 = 0$

$$a_{33} : -1 - \left(-\frac{2}{3}\right) \times 5 = \frac{7}{3}$$

$$a_{34} : -2 - \left(-\frac{2}{3}\right) \times 0 = -2$$

$$a_{35} : -1 - \left(-\frac{2}{3}\right) \times -5 = -\frac{13}{3}$$

$$a_{36} : 2 - \left(-\frac{2}{3}\right) \times -1 = \frac{4}{3}$$

$$a_{37} : -19 - \left(-\frac{2}{3}\right) \times 12 = -11$$

To make $a_{42} = 0$

Row 4 - $\left(\frac{a_{42}}{a_{22}}\right) \times \text{Row 2}$; But $\frac{a_{42}}{a_{22}} = \frac{-3}{-3} = 1$

$$\text{For } a_{42} : -3 - (1) \times -3 = 0$$

$$a_{43} : 9 - (1) \times 5 = 4$$

$$a_{44} : -6 - (1) \times 0 = -6$$

$$a_{45} : -13 - (1) \times -5 = -8$$

$$a_{46} : 6 - (1) \times -1 = 7$$

$$a_{47} : -23 - (1) \times 12 = -35$$

To make $a_{52} = 0$

Row 5 - $\left(\frac{a_{52}}{a_{22}}\right) \times \text{Row 2}$; But $\frac{a_{52}}{a_{22}} = \frac{-2}{-3} = \frac{2}{3}$

$$\text{For } a_{52} : 2 - \left(\frac{2}{3}\right) \times -3 = 0$$

$$a_{53} : -4 - \left(\frac{2}{3}\right) \times 5 = -\frac{24}{3}$$

$$a_{54} : 6 - \left(\frac{2}{3}\right) \times 0 = 6$$

$$a_{55} : 10 - \left(\frac{2}{3}\right) \times -5 = \frac{20}{3}$$

$$a_{56} : 0 - \left(\frac{2}{3}\right) \times -1 = \frac{2}{3}$$

$$a_{57} : 28 - \left(\frac{2}{3}\right) \times 12 = 36$$

To make $a_{62} = 0$

Row 6 - $\left(\frac{a_{62}}{a_{22}}\right) \times \text{Row 2}$; But $\frac{a_{62}}{a_{22}} = \frac{-1}{-3} = \frac{1}{3}$

$$\text{For } a_{62} : -1 - \left(\frac{1}{3}\right) \times -3 = 0$$

$$a_{63} : 9 - \left(\frac{1}{3}\right) \times 5 = \frac{22}{3}$$

$$a_{64} : -10 - \left(\frac{1}{3}\right) \times 0 = -10$$

$$a_{65} : -15 - \left(\frac{1}{3}\right) \times -5 = -\frac{40}{3}$$

$$a_{66} : 2 - \left(\frac{1}{3}\right) \times -1 = \frac{7}{3}$$

$$a_{67} : -43 - \left(\frac{1}{3}\right) \times 12 = -47$$

⑤

stage 2

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & 1 & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & 1 & -35 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 & 1 & 36 \\ 0 & 0 & 22/3 & -10 & -40/3 & 7/3 & 1 & -47 \end{bmatrix}$$

To make $a_{43} = 0$
 Row 4 - $\left(\frac{a_{43}}{a_{33}}\right) \times \text{Row 3}$; But $\frac{a_{43}}{a_{33}} = 4 \div \frac{7}{3} = \frac{12}{7}$

For a_{43} : $4 - \left(\frac{12}{7}\right) \times \frac{7}{3} = 0$
 a_{44} : $-6 - \left(\frac{12}{7}\right) \times -2 = -18/7$
 a_{45} : $-8 - \left(\frac{12}{7}\right) \times -13/3 = -4/7$
 a_{46} : $7 - \left(\frac{12}{7}\right) \times 4/3 = 33/7$
 a_{47} : $-35 - \left(\frac{12}{7}\right) \times -11 = -113/7$

To make $a_{53} = 0$
 Row 5 - $\left(\frac{a_{53}}{a_{33}}\right) \times \text{Row 3}$; But $\frac{a_{53}}{a_{33}} = \frac{-2}{3} \div \frac{7}{3} = -2/7$

For a_{53} : $-2/3 - \left(-\frac{2}{7}\right) \times \frac{7}{3} = 0$
 a_{54} : $6 - \left(-\frac{2}{7}\right) \times -2 = 38/7$
 a_{55} : $20/3 - \left(-\frac{2}{7}\right) \times -13/3 = 38/7$
 a_{56} : $-2/3 - \left(-\frac{2}{7}\right) \times 4/3 = -2/7$
 a_{57} : $36 - \left(-\frac{2}{7}\right) \times -11 = 230/7$

To make $a_{63} = 0$
 Row 6 - $\left(\frac{a_{63}}{a_{33}}\right) \times \text{Row 3}$; But $\frac{a_{63}}{a_{33}} = \frac{22}{3} \div \frac{7}{3} = \frac{22}{7}$

For a_{63} : $22/3 - \left(\frac{22}{7}\right) \times \frac{7}{3} = 0$
 a_{64} : $-10 - \left(\frac{22}{7}\right) \times -2 = -26/7$
 a_{65} : $-\frac{40}{3} - \left(\frac{22}{7}\right) \times -13/3 = 2/7$
 a_{66} : $7/3 - \left(\frac{22}{7}\right) \times 4/3 = -13/7$
 a_{67} : $-47 - \left(\frac{22}{7}\right) \times -11 = -87/7$

6

Stage 3 (Forward Elimination)

$$\begin{bmatrix} 1 & 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 38/7 & 38/7 & -2/7 & 230/7 \\ 0 & 0 & 0 & -26/7 & 2/7 & -13/7 & -87/7 \end{bmatrix}$$

To make $a_{54} = 0$

Row 5 - $\left(\frac{a_{54}}{a_{44}}\right) \times \text{Row 4}$; But $\frac{a_{54}}{a_{44}} = \frac{38}{7} \div \frac{-18}{7} = -\frac{19}{9}$

For $a_{54} \div$ $38/7 - (-19/9) \times -18/7 = 0$
 $a_{55} \div$ $38/7 - (-19/9) \times -4/7 = 38/9$
 $a_{56} \div$ $-2/7 - (-19/9) \times 33/7 = 29/3$
 $a_{57} \div$ $230/7 - (-19/9) \times -113/7 = -11/9$

To make $a_{64} = 0$

Row 6 - $\left(\frac{a_{64}}{a_{44}}\right) \times \text{Row 4}$; But $\frac{a_{64}}{a_{44}} = \frac{-26}{7} \div \frac{-18}{7} = \frac{13}{9}$

For $a_{64} \div$ $-26/7 - (13/9) \times -18/7 = 0$
 $a_{65} \div$ $2/7 - (13/9) \times -4/7 = 10/9$
 $a_{66} \div$ $-13/7 - (13/9) \times 33/7 = -26/3$
 $a_{67} \div$ $-87/7 - (13/9) \times -113/7 = 98/9$

Stage 4

$$\begin{bmatrix} 1 & 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 10/9 & -26/3 & 98/9 \end{bmatrix}$$

To make $a_{65} = 0$

Row 6 - $\left(\frac{a_{65}}{a_{55}}\right) \times \text{Row 5}$; But $\frac{a_{65}}{a_{55}} = \frac{10}{9} \div \frac{38}{9} = \frac{10}{38}$

⑦

$$\begin{aligned} \text{For } a_{65} &: \frac{10}{9} - \left(\frac{10}{38}\right) \times \frac{38}{9} = 0 \\ a_{66} &: \frac{-26}{3} - \left(\frac{10}{38}\right) \times \frac{29}{3} = -\frac{213}{19} \\ a_{67} &: \frac{98}{9} - \left(\frac{10}{38}\right) \times \frac{-11}{9} = \frac{213}{19} \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 1 & -213/19 & 213/19 \end{bmatrix}$$

Back substitution

$$\bar{T}_6 : 0\bar{T}_1 + 0\bar{T}_2 + 0\bar{T}_3 + 0\bar{T}_4 + 0\bar{T}_5 + -\frac{213}{19}\bar{T}_6 = \frac{213}{19}$$

$$-\frac{213}{19}\bar{T}_6 = \frac{213}{19}$$

$$\bar{T}_6 = \frac{213}{19} \div \frac{-213}{19} = \frac{213}{19} \times \frac{19}{-213}$$

$$\bar{T}_6 = -1$$

$$\bar{T}_5 : 0\bar{T}_1 + 0\bar{T}_2 + 0\bar{T}_3 + 0\bar{T}_4 + \frac{38}{9}\bar{T}_5 + \frac{29}{3}\bar{T}_6 = -\frac{11}{9}$$

$$\frac{38}{9}\bar{T}_5 + \frac{29}{3}(-1) = -\frac{11}{9}$$

$$\bar{T}_5 = \left(\frac{-11}{9} + \frac{29}{3}\right) \div \frac{38}{9} = \frac{76}{9} \times \frac{9}{38} = 2$$

$$\bar{T}_5 = 2$$

$$\bar{T}_4 : 0\bar{T}_1 + 0\bar{T}_2 + 0\bar{T}_3 + \left(-\frac{18}{7}\right)\bar{T}_4 - \frac{4}{7}\bar{T}_5 + \frac{33}{7}\bar{T}_6 = -\frac{113}{7}$$

$$-\frac{18}{7}\bar{T}_4 - \frac{4}{7}(2) + \frac{33}{7}(-1) = -\frac{113}{7}$$

$$\bar{T}_4 = \left(-\frac{113}{7} + \frac{8}{7} + \frac{33}{7}\right) \div -\frac{18}{7}$$

$$\bar{T}_4 = \frac{-72}{18} \times \frac{7}{-18} = 4$$

(8)

$$T_3: 0T_1 + 0T_2 + \frac{7}{3}T_3 - 2T_4 - \frac{13}{3}T_5 + \frac{4}{3}T_6 = -11$$

$$\frac{7}{3}T_3 - 2(4) - \frac{13}{3}(2) + \frac{4}{3}(-1) = -11$$

$$T_3 = \left(-11 + 8 + \frac{26}{3} + \frac{4}{3} \right) \div \frac{7}{3}$$

$$T_3 = \frac{7 \times 3}{1} = 3$$

$$T_2: 0T_1 + (-3)T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12$$

$$-3T_2 + 5T_3 - 5T_5 - (-1) = 12$$

$$T_2 = \left(12 - 5(3) + 5(2) - 1 \right) \div -3$$

$$T_2 = 6 \div -3 = -2$$

$$T_1: T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 = 4 + (-2) + 2(3) - 4 - 3(2) + (-1)$$

$$T_1 = 4 + 2 + 6 - 4 - 6 - 1$$

$$T_1 = 1$$

Therefore, the values of the temperatures are

$$T_1 = 1^\circ\text{C}$$

$$T_2 = -2^\circ\text{C}$$

$$T_3 = 3^\circ\text{C}$$

$$T_4 = 4^\circ\text{C}$$

$$T_5 = 2^\circ\text{C}$$

$$T_6 = -1^\circ\text{C}$$