Bullen Florence Ikeh - Ochuwah
16/ENG01/005
Chemical Engineering

ENG 382 - Engineering Mathematics IV

Assignment 2

If the maximum Percentage absolute error is desired to be 1E-9, using the newton-Raphson Iteration method and initial guess value of 0.5, find the root of the function given below;

$$f(x) = e^{-0.5x}(4-x) - 2$$

Solution

$$f(x) = e^{-0.5x}(4-x) - 2$$

$$f'(x) = -e^{-0.5x} - 0.5e^{-0.5x}(4-x)$$

$$f'(x) = -e^{-0.5x} - 2e^{-0.5x} + 0.5xe^{-0.5x}$$

$$f'(x) = -3e^{-0.5x} + 0.5xe^{-0.5x}$$

$$\frac{f(x)}{f'(x)} = \frac{e^{-0.5x}(4-x) - 2}{0.5xe^{-0.5x} - 3e^{-0.5x}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

when $x = 0.5$

$$x_1 = 0.5 - \left[\frac{e^{-0.5\times0.5}(4-0.5) - 2}{(0.5\times0.5e^{-0.5\times0.5}) - (3e^{-0.5\times0.5})}\right]$$

$$x_1 = 0.838890606$$

when $x = 0.838890606$

$$x_2 = 0.838890606 - \left[\frac{e^{-0.5\times0.838890606}(4-0.838890606) - 2}{(0.5\times0.838890606e^{-0.5\times0.838890606}) - (3e^{-0.5\times0.838890606})}\right]$$

$$x_2 = 0.884956003$$

when $x = 0.884956003$

$$x_3 = 0.884956003 - \left[\frac{e^{-0.5\times0.884956003}(4-0.884956003) - 2}{(0.5\times0.884956003e^{-0.5\times0.884956003}) - (3e^{-0.5\times0.884956003})}\right]$$

$$x_3 = 0.885708605$$

when $x = 0.885708605$

$$x_4 = 0.885708605 - \left[\frac{e^{-0.5\times 0.885708605}(4 - 0.885708605) - 2}{0.5\times 0.885708605\, e^{-0.5\times 0.885708605} - \left(3e^{-0.5\times 0.885708605}\right)}\right]$$

$$x_4 = 0.885708802$$

when $x = 0.885708802$

$$x_5 = 0.885708802 - \left[\frac{e^{-0.5\times 0.885708802}(4 - 0.885708802) - 2}{0.5\times 0.885708802\, e^{-0.5\times 0.885708802} - \left(3e^{-0.5\times 0.885708802}\right)}\right]$$

$$x_5 = 0.885708802$$

when $x = 0.885708802$

$$x_6 = 0.885708802 - \left[\frac{e^{-0.5\times 0.885708802}(4 - 0.885708802) - 2}{0.5\times 0.885708802\, e^{-0.5\times 0.885708802} - \left(3e^{-0.5\times 0.885708802}\right)}\right]$$

$$x_6 = 0.885708802$$

for Error

$$\varepsilon_{a} = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100\%$$

$$\varepsilon_{a_1} = \frac{0.838890606 - 0.5}{0.8} \times 100\%$$

$$\varepsilon_{a_1} = 40.397472994\%$$

$$\varepsilon_{a_2} = \frac{0.884956003 - 0.838890606}{0.884956003} \times 100\%$$

$$\varepsilon_{a_2} = 5.205388097\%$$

$$\varepsilon_{a_3} = \frac{0.885708605 - 0.884956003}{0.885708605} \times 100\%$$

$$\varepsilon_{a_3} = 0.084972043\%$$

$$\varepsilon_{a4} = \frac{0.885708802 - 0.885708605}{0.885708802} \times 100\%$$

$$\varepsilon_{a4} = 2.224207319 \times 10^{-5}$$

$$\varepsilon_{a5} = \frac{0.885708802 - 0.885708802}{0.885708802} \times 100\%$$

$$\varepsilon_{a5} = 0\%$$

i	x	ε_a
0	0.5	40.39747299
1	0.838890606	5.205388097
2	0.884956003	0.089972093337
3	0.885708605	$2.224207319 \times 10^{-5}$
4	0.885708802	0
5	0.885708802	0