

ASSIGNMENT III

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16/ENG 06/063

MECHANICAL ENGINEERING

ENG 382: ENGINEERING MATHEMATICS IV

Question:

If the model of a system having thermocouples measuring temperatures, $T(^{\circ}\text{C})$, at its different point is given by the set of expressions in Equation (1), estimate the values of the temperatures using:

- Gauss elimination method manually (with the aid of calculator),
- Gauss elimination method with the aid of Microsoft Excel,
- Gauss elimination method with the aid of MATLAB,
- Matrix inverse method with the aid of Microsoft Excel, and
- Matrix inverse method with the aid of MATLAB

$$\begin{cases} T_1 + T_2 - 2T_3 + 3T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{cases}$$

Solution

a) The augmented matrix (1)

$$\tilde{A}^{(1)} = \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$\tilde{A}^{(2)} = \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ \frac{E_2}{2} - E_1 \Rightarrow & 0 & -1.5 & 2.5 & 0 & -2.5 & -0.5 & 6 \\ E_3 - E_1 \Rightarrow & 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ \frac{E_4}{5} - E_1 \Rightarrow & 0 & -0.6 & 1.8 & -1.2 & -2.6 & 1.2 & -4.6 \\ \frac{E_5}{-3} - E_1 \Rightarrow & 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ \frac{E_6}{4} - E_1 \Rightarrow & 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -10.75 \end{array} \right]$$

Divide through E_3 by 2 and swap (pivot) with E_2

$$\tilde{A}^{(2)} = \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & -1.5 & 2.5 & 0 & -2.5 & -0.5 & 6 \\ 0 & -0.6 & 1.8 & -1.2 & -2.6 & 1.2 & -4.6 \\ 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -10.75 \end{array} \right]$$

$$A^{(3)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ \frac{E_3}{1.5} + E_2 \Rightarrow & 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -5.5 \\ \frac{E_4}{0.6} + E_2 \Rightarrow & 0 & 0 & 2.5 & -3 & -4.833 & 3 & -17.167 \\ \frac{E_5}{0.667} + E_2 \Rightarrow & 0 & 0 & 1.499 & -3.999 & -5.497 & 1 & -23.493 \\ \frac{E_6}{0.25} + E_2 \Rightarrow & 0 & 0 & 8.5 & -11 & -15.5 & 3 & -52.5 \end{bmatrix}$$

Divide through E_4 by 2.5 and swap with E_3

$$A^{(3)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -5.5 \\ 0 & 0 & 1.499 & -3.999 & -5.497 & 1 & -23.493 \\ 0 & 0 & 8.5 & -11 & -15.5 & 3 & -52.5 \end{bmatrix}$$

$$A^{(4)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ \frac{E_4}{1.167} - E_3 \Rightarrow & 0 & 0 & 0 & 0.343 & 0.076 & -0.628 & 2.154 \\ \frac{E_5}{1.499} - E_3 \Rightarrow & 0 & 0 & 0 & -1.468 & -1.784 & -0.533 & -8.805 \\ \frac{E_6}{8.5} - E_3 \Rightarrow & 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{bmatrix}$$

Divide through E_4 by 0.343

$$A^{(4)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & -1.468 & -1.734 & -0.533 & -8.805 \\ 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{bmatrix}$$

$$A^{(5)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 0.959 & 2.194 & -0.282 \\ 0 & 0 & 0 & 0 & -1.382 & 10.842 & -13.631 \end{bmatrix}$$

$\frac{E_5}{-1.468} - E_4$
 $\frac{E_6}{-0.094} - E_4$

Divide through E_5 by 0.959

$$A^{(5)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & -1.382 & 10.842 & -13.631 \end{bmatrix}$$

$$\tilde{A}^{(6)} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.983 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & 0 & -10.133 & 10.157 \end{bmatrix}$$

$\frac{E_6}{-1.382} - E_5$

Backward Substitution

$$\tilde{A}^{(6)} \text{ from } E_6 ; -10.133 T_6 = 10.157$$

$$T_6 = -1.002 \approx -1 //$$

$$\text{from } E_5 ; T_5 + 2.288 T_6 = -0.294$$

$$T_5 = -0.294 - 2.288(-1.002)$$

$$T_5 = 1.999 \approx 2 //$$

$$\text{from } E_4 ; T_4 + 0.222 T_5 - 1.831 T_6 = 6.280$$

$$T_4 = -0.222(1.999) + 1.831(-1.002) + 6.280$$

$$T_4 = 4.002 \approx 4 //$$

$$\text{from } E_3 ; T_3 - 1.2 T_4 - 1.933 T_5 + 1.2 T_6 = -6.867$$

$$T_3 = +1.2(4.002) + 1.933(1.999) - 1.2(-1.002) - 6.867$$

$$T_3 = 3.002 \approx 3 //$$

$$\text{from } E_2 ; T_2 - 0.5 T_3 - T_4 - 0.5 T_5 + T_6 = -9.5$$

$$T_2 = 0.5(3.002) + 4.002 + 0.5(1.999) + (-1.002) - 9.5$$

$$T_2 = -1.996 \approx -2 //$$

from ϵ_1 ;

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 = 4 - (-1.996) + 2(3.002) - 4.002 - 3(1.999) + (-1.002)$$

$$T_1 = 0.999 \approx 1 //$$

$$\therefore T_1 = 1, T_2 = -2, T_3 = 3$$

$$T_4 = 4, T_5 = 2 \text{ \& } T_6 = -1$$

b.) Exec file attached to Zip folder.

c.) Matlab file attached to Zip folder.

MATLAB PROGRAM CODE

```
1 function C = assign3(A,B)
2- A = [1 1 -2 1 3 -1; 2 -1 1 2 1 -3; 1 3 -3 -1 2 1;
3- 5 2 -1 -1 2 1; -3 -1 2 3 1 3; 4 3 1 -6 -3 -2];
3- B = [4; 20; -15; -3; 16; -27];
45- i = 1;
56- x = [A B];
67- [m n] = size(x);
78- while i <= m
89-     if X(i,i) == 0
```



```

9-      disp('Diagonal element zero')
10-      return
11-      end
12-      X = elimination(X, i, i);
13-      i = i + 1;
14-  - end
15-  - C = X(:, n);

16-  function X = elimination(X, i, j)
17-      [m n] = size(X);
18-      a = X(i, j);
19-      X(i, :) = X(i, :) / a;
20-      for k = 1:m
21-          if k == i
22-              continue
23-          end
24-          X(k, :) = X(k, :) - X(i, :) * X(k, j);
25-      end

```

d.) Excel file attached to Zip folder.

8- Matlab file attached to zip file

MATLAB PROGRAM CODE.

~~1- Command window~~

1- Command window

2- clear

3- clc

4- close all

5- $A = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix}$

6- $x = [4; 20; -15; -3; 16; -27]$

7- $n = \text{inv}(A)$

8- $b = \text{inv}(A) * x$